

# **DYNAMIC ANALYSIS OF A WIND TURBINE BLADE**

By

**SAMBIT SARANGI**

**110ME0330**



**Department of Mechanical Engineering  
National Institute of Technology  
Rourkela 769008**

# **DYNAMIC ANALYSIS OF A WIND TURBINE BLADE**

*A Thesis submitted in partial fulfillment  
of the requirements for the degree of*

**Bachelor of Technology  
In  
Mechanical Engineering**

By

**Sambit Sarangi (110ME0330)**

Under the guidance of

**Dr. R. K. Behera**



**Department of Mechanical Engineering  
National Institute of Technology  
Rourkela 769008**



## National Institute of Technology Rourkela

### CERTIFICATE

This is to certify that the thesis entitled, “**DYNAMIC ANALYSIS OF A WIND TURBINE BLADE**” submitted by **Sambit Sarangi (110ME0330)** in partial fulfillment of the requirements for the award of **Bachelor of Technology Degree** in **Mechanical Engineering** at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

Dr. R.K. Behera

Dept. of Mechanical Engineering

National Institute of Technology

Rourkela-769008

Date:

## ACKNOWLEDGEMENT

I am highly indebted to my mentor **Dr. R. K. Behera** for providing me the opportunity to work on this project under his able guidance. He has been great as a mentor and has motivated me thorough out my entire work duration. I am thankful to him for being so patient with me and for providing me with the necessary tools, knowledge and help to complete this project.

I am also grateful to **Mr. Pawan Kumar** of Mechanical Department for helping me with concepts and ideas in the initial days of my project work. I am also thankful to all the teachers and staffs of the Department of Mechanical Engineering for always being available to me for clearing some doubts or other issues which was hindering my progress in my project work.

Last but not the least, I humbly thank the Almighty for giving me such wonderful parents and sibling and for endowing me with the necessary skills to brave all odds and emerge victorious. I am grateful to my family for constantly supporting me in hard times and praying for me.

Date:

SAMBIT SARANGI  
(110ME0330)  
B. Tech,  
Mechanical Engineering  
NIT, Rourkela

## **ABSTRACT**

The world is running out of conventional energy sources and there is a pressing need of utilizing non-traditional energy sources to endure the ever escalating energy needs. Wind turbines provide an alternative way of generating energy from the power of wind. At windy places, wind speeds can achieve scintillating values of 10-12 m/s. Such high speeds of wind can be utilized to harness energy by installing a wind turbine usually having 3 blades. The geometry of the blades is made as such that it generates lift from the wind and thus rotates. The lift force generates a moment around the hub and thus the combined torque effort of 3 blades rotates the turbine and generates electricity. Rotational speed of the blades is usually 6 times that of wind speed. In this project, validation of a beam (a geometrical approximation of a blade) in vibration analysis is taken up first. The natural frequencies are matched with a published research paper and then an actual blade geometry is taken up to validate its 1<sup>st</sup> 3 natural frequencies with a published research paper and then a CFD analysis is taken up to find the lift and drag forces on the blade and subsequently these forces are used to calculate the fatigue life of the blade. Suitable materials for different parts of the blade are taken to see which combination of materials gives better results.

# CONTENTS

CERTIFICATE .....	i
ACKNOWLEDGEMENT .....	ii
ABSTRACT .....	iii
LIST OF FIGURES .....	vii
LIST OF TABLES .....	viii
1. INTRODUCTION .....	1
2. WIND ENERGY .....	3
2.1. WIND AS AN ENERGY SOURCE.....	3
2.2. WIND POWER IN INDIA .....	4
2.3. WIND TURBINES .....	5
2.4. WORKING PRINCIPLE.....	5
2.5. DESIGN OF WIND TURBINES.....	6
2.6. AERODYNAMICS .....	6
2.7. POWER CONTROL .....	7
2.8. Stall.....	7
2.9. Pitch control.....	8
3. AIM OF THE PRESENT WORK .....	9
4. LITERATURE REVIEW .....	10
5. METHODOLOGY ADOPTED .....	14

5.1.	<i>GEOMETRIC MODELING</i> .....	15
5.1.1.	Modeling of Timoshenko beam as an approximation to the windmill blade: .....	15
5.1.2.	Modeling of actual windmill blade geometry:.....	16
5.2.	<i>MODAL ANALYSIS</i> .....	20
5.2.1.	Modal analysis of Timoshenko beam as specified in [7]:.....	20
5.2.2.	Modal Analysis of actual windmill blade model [8]: .....	21
5.3.	<i>CFD ANALYSIS</i> .....	25
5.4.	<i>STRESS ANALYSIS</i> .....	26
5.5.	<i>FATIGUE LIFE ANALYSIS</i> .....	27
6.	<i>MATERIALS USED</i> .....	28
7.	<i>BLADE GEOMETRIC PARAMETERS</i> .....	32
8.	<i>RESULTS AND DISCUSSIONS</i> .....	33
8.1.	<i>MODAL ANALYSIS OF BEAM</i> .....	33
8.2.	<i>MODAL ANALYSIS OF TRUE BLADE MODEL</i> .....	36
8.3.	<i>CFD ANALYSIS OF TRUE BLADE MODEL IN FLUENT</i> .....	38
8.3.1.	Pressure contours .....	39
8.3.2.	Velocity vector field .....	40
8.4.	<i>STRESS ANALYSIS</i> .....	43
8.5.	<i>FATIGUE LIFE PREDICTION</i> .....	44
9.	<i>CONCLUSION</i> .....	45
10.	<i>REFERENCES</i> .....	46

## LIST OF FIGURES

Figure 1. Model of a beam.....	16
Figure 2. Cross section of windmill blade as per NACA 4415 airfoil.....	17
Figure 3. CATIA sketch of blade cross section. ....	17
Figure 4. Blade geometry as per ref. [8] .....	18
Figure 5. CATIA part corresponding to the above geometry. ....	19
Figure 6. Meshed model of beam as per ref. [7].....	20
Figure 7. Boundary condition for the beam. ....	21
Figure 8. Actual blade geometry after importing to ANSYS. ....	22
Figure 9. Blade geometry with skin hidden to show inner parts. ....	22
Figure 10. Meshed interior of the blade.....	23
Figure 11. Meshed view of the blade with skin.....	24
Figure 12. Boundary condition for the blade model.....	24
Figure 13. FEV model of blade in FLUENT as wireframe. ....	25
Figure 14. FEV model of blade in FLUENT as surfaceframe.....	26
Figure 15. Boundary condition of the blade for stress analysis.....	27
Figure 16. Fatigue load case. ....	28
Figure 17. S-N curve of structural steel.....	30
Figure 18. S-N curve for epoxy-S-glass. ....	30
Figure 19. S-N curve for Epoxy-carbon UD 230Gpa. ....	31
Figure 20. Reference curves for various useful materials [9].....	31



Figure 21. Mode shape of blade corresponding to 1st modal frequency. ....	33
Figure 22. Mode shape of blade corresponding to 2nd modal frequency. ....	34
Figure 23. Mode shape of blade corresponding to 3rd modal frequency. ....	34
Figure 24. Mode shape of blade corresponding to 4th modal frequency. ....	35
Figure 25. 1st mode shape of blade. ....	36
Figure 26. 2nd mode shape of blade. ....	37
Figure 27. 3rd mode shape of blade. ....	37
Figure 28. Pressure contour on upper skin of blade. ....	39
Figure 29. Pressure contour on lower skin of blade. ....	39
Figure 30. Velocity vector field on upper skin of blade. ....	41
Figure 31. Velocity vector field on lower skin of blade. ....	41
Figure 32. Hotspots of the blade geometry. ....	43
Figure 33. Deformation analysis of the windmill blade. ....	43

## LIST OF TABLES

Table 1. Sectional characteristics of the blade span wise. ....	16
Table 2. Material properties as specified in ref. [8] .....	28
Table 3. Material property for Epoxy-S-glass UD material. ....	29
Table 4. Material properties for Epoxy-carbon UD 230Gpa. ....	29
Table 5. Geometric properties of Blade. ....	32
Table 6. Blade components geometric and physical parameters. ....	32
Table 10. Listing of drag force for the blade model. ....	42
Table 11. Listing of lift force values for the blade model. ....	42
Table 12. Fatigue life prediction analysis of the blade model. ....	44

# 1. INTRODUCTION

A lot of literature in the form of research papers, journals, patents etc. is available where one can find that substantial work, theoretical and computational, has been done on the dynamic vibration analysis of beams [1-3]. But fatigue analysis of a proper bladed structure is a very rare occurrence and only a handful of literature is available on it. Vibration analysis is a very useful technique when it comes to analyzing the dynamics of a structure. It provides data about the natural frequencies, mode shapes and frequency response function of vibration. The natural frequencies are critical to a given structure. If a force with a similar frequency is applied to the structure, it can cause catastrophic failure by increasing the deflections many folds. Not just the frequency, but the mode shape of the structure also has to be replicated by the propagation of the applied force. Only when both, natural frequency and mode shape, match to applied frequency and propagation of force, then true resonance occurs and the material undergoes severe failure. Thus the structure must be designed to avoid the occurrence of resonance. And if resonance cannot be avoided, then it should have a facility of damping it away so as to cause minimum damage to the components. Many cases are present where avoiding resonance is not an option, e.g. a propeller blade has to pass through a number of resonant frequencies starting from 0 Hz as it rotates at a very high speed. Thus mechanisms are designed to wither away the effect of passing resonance in such machines/components. A comprehensive stress analysis is also a very necessary tool to identify the hotspots in the geometry and carryout further reinforcing or design changes. Fatigue analysis is the most important analysis as far as mechanical devices subjected to varying service loads are concerned. Any component when subjected to repeated loading undergoes a series of molecular deformations adding up to a kind of failure termed as fatigue failure. In brief, it is the gradual ageing of the component due to forces and stresses and other

environmental factors. A fatigue analysis is one of the most difficult and challenging analysis there is. It requires a lot of design data and a lot of experimental work to validate the results. A true design report is complete in useful sense only when it can co-relate the vibration characteristics to the applied forces, induced stresses and finally give as output- the service life of the component. Better yet if it can also give a cost analysis and optimization of the parameters to produce an economic design. Although these topics individually provide great insight into specific areas of necessary research work, yet they somehow seem incomplete if viewed separately. Therefore an attempt has been made in this research paper to bring together these subtopics into a common heading and thus make the design report seem complete from a design point of view.

## **2. WIND ENERGY**

### ***2.1.WIND AS AN ENERGY SOURCE***

Wind power has been used by mankind ever since they have known to put sails to their boats and canoes. For a lot of time now wind driven machines have been grinding grains and pumping water. This energy form has always been extensively available and not limited to river banks or sources of fuel. With the advancements in electricity, wind power has found renewed usages in powering buildings remotely from a centrally generated power source. This day, wind powered generators come in wide size ranges that can charge batteries or meet electricity need of a large population [4].

Wind power is the manifestation of converting energy from wind into other useful forms of energy with the help of wind turbines for making electrical power, windmills for mechanical power, wind pumps for water pumping activities/drainage, or sails for propelling ships.

Large wind farms have numerous individual wind turbines that are connected to the electric power grid. Onshore wind is quite an inexpensive source of electricity generation, competitive with or even cheaper than fossil fuel plants. Small-scale onshore wind farms give electricity to isolated habitats. Some companies buy surplus electricity produced by small-scale domestic wind turbines. Offshore locations for windmills are better than land based locations due to presence of steadier and stronger winds. Only thing of worry is that maintenance costs are quite high.

Wind power has been pretty consistent for years but has considerable deviations over shorter time scales. If the proportion of wind power in a region heightens, a necessity to upgrade the grid and lower the ability of conventional production could occur. Power management techniques like having excess capacity storage, topologically distributed turbines, dispatch-able power backing

sources, storage like pumped-storage hydel, exchanging power to neighboring areas or reducing demand when wind speed is low, can largely solve these problems. Weather forecasting also helps permit the electricity network to get ready for the predictable variations in production [4].

## ***2.2.WIND POWER IN INDIA***

The 1990s saw an upsurge in the development of wind power in the democratic republic of India. India is ranked fifth in the world in terms of installed wind power capacity.

As of 31 December 2013 the installed capacity of wind power in India was 20149 MW, mainly spread across Tamil Nadu (7154 MW), Gujarat (3093 MW), Maharashtra (2976 MW), Karnataka (2113 MW), Rajasthan (2355 MW), Madhya Pradesh (386 MW), Andhra Pradesh (435 MW), Kerala (35.1 MW), Orissa (2MW), West Bengal (1.1 MW) and other states (3.20 MW). It is estimated that 6,000 MW of additional wind power capacity will be installed in India by 2014. Wind power accounts for 8.5% of India's total installed power capacity, and it generates 1.6% of the country's power [4].

A target of 10,500 MW between 2007 and 2012 has been fixed by The Ministry of New and Renewable Energy (MNRE), but an extra generation facility of only about 6,000 MW possibly will be available for profitable use by 2012. The MNRE has announced a revised estimation of the potential wind resource in India from 49,130 MW assessed at 50m Hub heights to 102,788 MW assessed at 80m Hub height. The wind resource is possibly even greater for contemporary wind turbines at higher Hub heights.

### ***2.3.WIND TURBINES***

The device that is used to generate this non-conventional and clean source of energy from wind is known as a Wind Turbine or Windmill. Extraction of energy from wind is the primordial function of wind turbines. Aerodynamics naturally is an important aspect related to wind turbines. These are of different types based on various energy extraction methods. Overall the aspects of aerodynamics depend largely on the geometry. However there are some fundamental concepts that can be applied to all turbines. Each topology has a limiting maximum power for a given flow, and certain topologies are superior to others. The method used to extract power has a strong influence on this. In general all turbines can be grouped as being lift based, or drag based with the former being more efficient. The difference between these groups is the aerodynamic force that is used to extract the energy.

The horizontal-axis wind turbine (HAWT) is the most commonly used topology. It is based on lift and gives very good performance and hence is commercially acceptable, thereby attractive much research activity [4].

### ***2.4.WORKING PRINCIPLE***

It consists of a hub which usually has 3 blades. There is an energy producing device, mostly a Dynamo, placed inside the hub that generates electricity when rotated by the blades. The blades are geometrically designed in such a way that when wind flows across them, it generates a lift force which acts at the center of pressure of the blade. This lift force produces a turning moment about the hub and the 3 blades combined together produce sufficient torque to generate usable amount of electricity from the wind turbine.

## ***2.5.DESIGN OF WIND TURBINES***

Wind turbine design is merely the process of defining the form and stipulations of a wind turbine for extracting energy from wind. A wind turbine installation comprises of necessary systems required to capture the wind's power, point the turbine towards the wind, convert mechanical turning into electric power, and other arrangements to start, stop, and rheostat the turbine.

This paragraph contains the design of a horizontal axis wind turbines as the majority of marketable turbines use this design.

The physicist Albert Betz, in 1919, showed that for a theoretical ideal wind-energy extraction device, the laws of conservation of mass and energy allowed not more than  $16/27$  (59.3%) of the total kinetic energy of the wind to be utilized. This Betz' law limit is approached by most modern turbine designs which typically reach 70 to 80% of this given theoretical limit [4].

In addition to aerodynamic construct of the blades, design of a comprehensive wind power system needs to also address design of hub, controls, the generator, the supporting structure and also the foundation. Further design questions likewise arise when assimilating wind turbines into electrical power grids.

## ***2.6.AERODYNAMICS***

Aerodynamics of a wind turbine blade is a very complicated phenomenon. The air flows near and farther from the blade is very different to each other. The very method of functioning of the machinery is instrumental in this aerodynamic effect. It works by the deflection of wind. However the phenomena experienced by the rotors of wind turbine blades are very different to those of other aerodynamic structures known to man.



## ***2.7.POWER CONTROL***

Design of a wind turbine is such that it can produce maximum power for a wide range of wind speeds. The maximum wind speed that a wind turbine can be subjected is called the survival speed, above this speed limit survival ceases. Values of survival speeds for commercially available turbine blades are from 40 m/s (144 km/h, 89 MPH) to 72 m/s (259 km/h, 161 MPH). The most common survival speed is 60 m/s (216 km/h, 134 mph). Wind turbines normally have three modes of operation:

- i. Below rated wind speed operation
- ii. Around rated wind speed operation (usually at nameplate capacity)
- iii. Above rated wind speed operation

Power generation has to be limited if the rated wind speeds are exceeded. There are a lot of ways for achieving this.

A control system constitutes three basic elements namely: sensors to measure process variables, actuators to manipulate energy capture and component loading, and control algorithms to coordinate the actuators based on information gathered by the sensors [4].

## ***2.8.Stall***

Stalling functions by escalating the angle at which the relative wind impinges the blades (angle of attack), and it lessens the induced drag (drag associated with lift). It can be made to happen passively and thus it is simple (it increases automatically when the winds speed up), but it enlarges the cross-section of the blade face-on to the wind. A fully stalled turbine blade has the flat side of the blade facing absolutely into the wind, when stopped.

A fixed-speed HAWT fundamentally increases its angle of attack at greater wind speed as the blades get sped up. A natural strategy is to permit the blade to stall when wind speeds increase. This technique was effectively used on many primitive HAWTs. It was also observed that degree of blade pitch stirred to amplify audible noise levels.

Vortex generators may be consumed to control the lift traits of the blade. The Vortex Generators are located on the airfoil to improve the lift if they are located on the lower (flatter) surface and limit the highest lift when placed on the upper surface [5].

## ***2.9.Pitch control***

Furling takes place by the reduction of the angle of attack, which diminishes the drag that is induced from lift of rotor and also from the cross-section. Problem which poses major threat in the designing of wind turbines is increasing the response of the blades to furl or stall rapidly enough if strong wind causes spontaneous acceleration. A fully furled blade has its edge facing into the wind when stopped.

These loads can be largely reduced by making structures softer and/or flexible [6]. Accomplishment of this is done with downwind rotors and/or with help of blades that are curved and can twist almost naturally to lessen the angle of attack at greater wind speeds. Such kind of systems will mostly be nonlinear.

Today's turbines mostly furl their blades in enormous winds. As furling is an action against torque on blades, it thus requires a method of pitch angle control, that can be obtained with a slewing drive. Slewing drive accurately angles the windmill blade while enduring high torque loads. Many turbines utilize hydraulic systems. Such systems are normally loaded with spring, so that in case of hydraulic power failure, the blades furl automatically. Small-scale wind turbines (lesser than 50 kW) with variable-pitching normally use systems driven by centrifugal force,

either by flyweights or else geometric design, and engage no electric or hydraulic controls. However, the methods for comprehension of full-span blade pitch control necessitate to be developed in order to upsurge energy capture and alleviate fatigue loads [6].

### **3. AIM OF THE PRESENT WORK**

The aim of the present work is to model a proper geometry of a small scale wind turbine blade and carryout dynamic analysis of the structure. The dynamic analysis would involve validating the natural frequencies of the structure (first 3 frequencies only), CFD analysis of the blade geometry to calculate the center of pressure along with the lift and drag forces and then finally a stress analysis based on the calculated forces to determine the fatigue life of the component.

## 4. LITERATURE REVIEW

A detailed literature review was done on a lot of published research papers featuring in a wide range of journals. Some of those which have inspired this work in a more promising way have been critically analyzed and provided below.

*Gupta and Rao* [7] carried out an eigenvalue analysis of tapered and twisted Timoshenko beams. The angle of twist was varied linearly along the length of the beam. Width and depth was also varied linearly along the beam span. Vibration analysis was done on these doubly tapered and twisted beams by deriving stiffness and mass matrices of the beam element. They formed a mathematical displacement model of the assumed geometry. Deflections in X and Y axes were taken by considering bending and shear in corresponding planes. They assigned 16 degrees of freedom to the 3D model in displacement analysis. Then element stiffness matrix was derived by using the total strain energy equation [7]. Cross sectional area and moments of inertia were computed as a function of beam span and twist angle. The final equation was written in matrix form which became the element stiffness matrix of the beam. Next the element mass matrix was formed by taking into account the kinetic energy of the element including shear deformation and rotary inertia [7]. The resulting matrix was the element mass matrix of the Timoshenko beam. Boundary conditions were applied corresponding to cantilever type beams. The eigenvalues of the resulting stiffness and mass matrices were calculated. The effects of breadth and depth taper ratios on the natural vibrational frequencies were investigated. However the beam geometry was just a vague approximation of turbine blade geometry. The initial work in this current project has been towards validating the results of the aforesaid published journal by modeling and analysis through CAD/CAE software like CATIA and ANSYS. In this work, actual mini-turbine blade geometry has been modeled in CAD software and its vibrational analysis has been carried out.

*Perkins and Cromack* [8] took up the design, stress and vibration analysis of a true blade mini-model. They carried out an experimental work. They manufactured a small scale windmill blade model and carried out experiments on a vibration test rig. The model was 16 feet long, tapered and twisted structure with a blade stock at its root end. The hollow blade skin had a spar and an end stiffener within it to provide bending rigidity. The chosen materials were isotropic in nature and thus the structure was an isotropic composite geometry. They used a NACA- 4415 airfoil as the cross-section of the blade. The skin material used was a relatively low bending modulus composition fiberglass epoxy matrix whereas the spar was a relatively high bending modulus fiberglass epoxy matrix. The blade stock was surrounded by a steel sleeve. They also calculated bending stress distributions, deflections under loads, mass of blade, mass moment of inertia about axis of rotation and natural frequencies of vibration of the blade.

*Kong et al.* [9] undertook a step by step method to calculate the fatigue life of a windmill blade. Fatigue load and stress spectra were obtained for short period operation. Sample fatigue load spectrum in time order was obtained. The load spectrum was ordered as per damage causing potential. Cyclic loads by empirical equations with coefficients of variations were calculated. Finally the cyclic stress value was computed. The required design life was calculated by using suitable formulae and allowable fatigue stress was calculated. Finally evaluation of fatigue strength was done and final life of the product was obtained in terms of number of years of service. S-N curves were used for fatigue analysis and Palmgren and Miner's damage rule was used to calculate the effective life of the component. Since they used a full scale model, their forces were large enough to cause substantial fatigue damage to limit the life to 20 years approximately. The blade modeled in this current work is a small scale model and thus the loads

are not as great as it is in this published research paper. Nevertheless, a fatigue analysis and life prediction is undertaken to find out the service life of the component.

*Larsen et al.* [10] did a modal analysis of wind turbine to identify its natural frequencies, damping characteristics and mode shapes. They considered a lot of experimental procedures like impact modal testing technique which resolves flap-wise and edgewise translations and chord rotations and hydraulic shaker test rig setup. They used data acquisition systems to mount accelerometers and strain gauges on the blade to study its deflection characteristics at different excitation frequencies. The theory behind this is that maximum deflections occur during resonance only. After isolating the natural frequencies, they carried out an FEA on the blade model to validate their experimental findings. Good agreement was found between measured deflection directions and FEA based deflection directions. However large discrepancies were observed in results of secondary deflection directions, maybe, owing to experimental uncertainties or inefficient structural coupling. Compared to previous work, their experimental investigation was about comparing various experimental modal analysis techniques and consequently identifying the most appropriate of these techniques bearing in mind unbound expenses, time consumption, experimental uncertainty and resolution.

*Ganesan and Zabihollah* [11-12] in their formulation and parametric study of vibration analysis of tapered composite beams used a higher order finite element to solve a Finite Element Model to obtain natural frequencies and mode shapes of the beam under consideration. Their motto was to study the free un-damped vibrations of beam structures. They did analysis of externally tapered composite beams as well as mid-plane tapered and internally tapered composite beams. They formulated element matrices for the 3 different cases namely element stiffness matrices and element mass matrices. The coefficients of these matrices were derived by symbolic calculations in MATLAB. Numerical example of a uniform thickness composite beam

was considered for solving to give a clear cut analytical idea of the mathematical solution process. The differential equation of motion of the beams and their corresponding variation finite element formulation considering the tapered composite structure of beams was developed. A higher-order finite element model was developed based on classical laminated theory for investigating the vibration response of classic laminated tapered composite beams. Various types of tapered composite beams were investigated for their vibration response. The developed variation formulation was validated by considering some numerical examples and relating the results with exact solutions and/or Ritz method, where applicable.

*Ganesan et al.* [13] in their research paper on study of tapered laminated composite structures, have elucidated the advantages of dropping off some plies at discrete positions in the laminate to be the structural tailoring capabilities, damage forbearance and moreover their potential for creating substantial weight saving properties in the field of engineering applications. They have discussed various approaches to model and analyze inter-laminar response of tapered composite structures using finite and non-finite elements. A review of displacement based finite elements and hybrid finite elements are also provided. Stress-strength and fracture mechanics approaches are studied for contribution to delamination of composite laminates. They have analyzed that terminating plies to create tapered laminates often lead to geometric and material discontinuities. These discontinuities act as a vital source of delamination initiation as well as propagation. It was concluded that delamination naturally initiates from the taper root and material non-linearity must always be considered. Moreover detailed experimentation is needed on resin toughness and inter-laminar fracture strength and multiple delamination should be considered.

In this current work, effort has been made towards modeling the same blade geometry in CATIA and carrying out dynamic vibration analysis to validate the natural frequencies to those published

in the research paper. Further, the research paper does not give any insight into the life of the component. This issue has been taken up in this current work as fatigue life prediction of the windmill blade considering different materials for different blade parts as compared to those in the research paper.

## **5. METHODOLOGY ADOPTED**

The methodology of this work is divided into five parts namely;

- i. Geometric modeling,
- ii. Vibration analysis,
- iii. CFD analysis,
- iv. Stress based analysis and
- v. Fatigue analysis to predict service life of the component.

Computer Aided Three-dimensional Interactive Application (CATIA), which is a CAD software, is used to model the blade geometry of the wind turbine as specified in [8]. Modal analysis is a vital tool in identifying and eliminating the resonant frequencies that lead to severe fatigue damages in high/low cycle fatigue problems. Computational Fluid Dynamics gives us an idea about the various pressure forces like lift and drag acting on a component present in a fluid flow field. In this work, CFD tool of ANSYS i.e. FLUENT has been used to determine the lift and drag forces, their magnitude and directions. Also the center of pressure has been found out where these pressure forces act naturally. These values give us an idea about the pressure and force distribution on the blade. The output of the CFD analysis is used to conduct a stress analysis in ANSYS workbench to find out the hotspots in the designed



geometry. These stresses are taken up on a repetitive cycle for a fatigue analysis performed in the same workbench module.

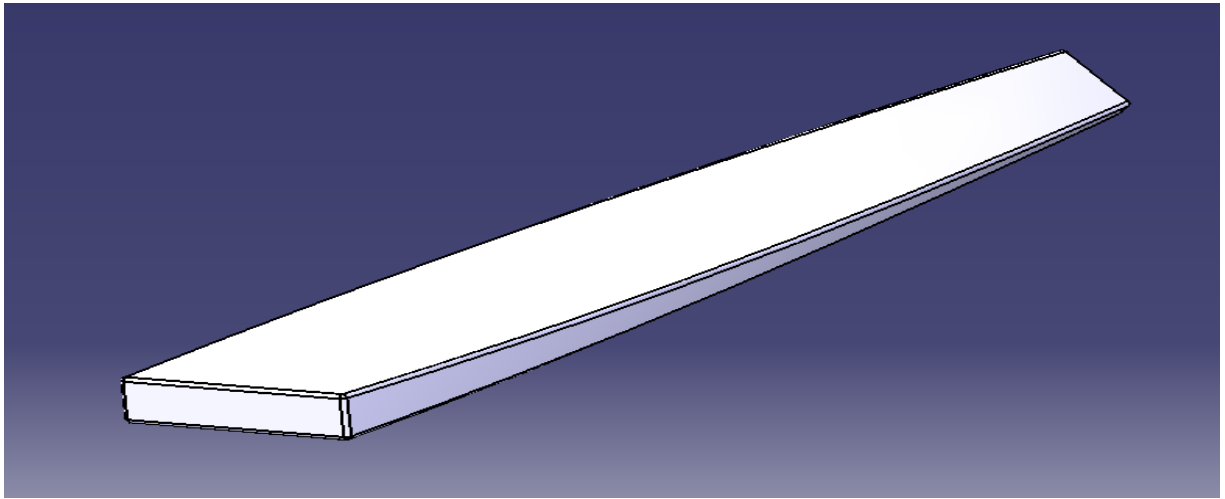
## ***5.1.GEOMETRIC MODELING***

### **5.1.1. Modeling of Timoshenko beam as an approximation to the windmill blade:**

The Timoshenko beam as specified in [7] was modeled in CATIA with the following design parameters as

- i. Length of beam = 0.1524 m
- ii. Breadth at root = 0.0254 m
- iii. Depth at root = 0.00635 m
- iv. Depth taper ratio = 2.29
- v. Breadth taper ratio = 2.56
- vi. Total linear twist = 45 deg.
- vii. Mass density =  $8000 \text{ kg/m}^3$
- viii. Young's modulus =  $2.07 \times 10^{11} \text{ N/m}^2$
- ix. Modulus of rigidity =  $7.7625 \times 10^{10} \text{ N/m}^2$

After modeling, the beam geometry looked like as shown in Figure 1.



**Figure 1. Model of a beam**

### **5.1.2. Modeling of actual windmill blade geometry:**

CATIA is used to model the complex blade geometry as specified in [8]. The mini-windmill blade has the following sectional characteristics as shown in Table 1.

**Table 1. Sectional characteristics of the blade span wise.**

<u>r/Radius (Station÷10)</u>	<u>Chord (ft)</u>	<u>Twist (degrees)</u>	<u>L.E. to Spar Web (ft)</u>	<u>Skin Thickness (in)</u>	<u>Spar Thickness (in)</u>	<u>Web Thickness (in)</u>
.1	1.35	45	.54	.036	.238	.238
.2	1.46	25.6	.584	.036	.238	.238
.3	1.26	15.9	.504	.036	.205	.220
.4	1.02	10.4	.41	.036	.205	.150
.5	.85	7.4	.34	.036	.205	.050
.6	.73	4.5	.29	.027	.186	.050
.7	.63	2.7	.25	.027	.148	.050
.8	.55	1.4	.22	.018	.110	.050
.9	.45	.4	.18	.018	.072	.050
1.0	.35	0	.14	.018	.053	.050

where 'r' is the radius of the blade section along the blade span from the root of the stock. Chord is the end to end length of the blade cross section. Twist is the progressive rotation of the blade cross-section about its axis so as to increase surface area for lift and drag forces. Skin is the outer covering of the blade, the one that imparts the NACA 4415 shape to it [14]. Spar web is the section provided inside the hollow skin to reinforce it with bending stiffness along with the spar. They constitute a U section. The trailing edge stiffener is provided to uphold the geometry at the end of the airfoil to prevent both faces from crimping and getting stuck to each other. Figure 2 shows a sketch of the blade section from [8].

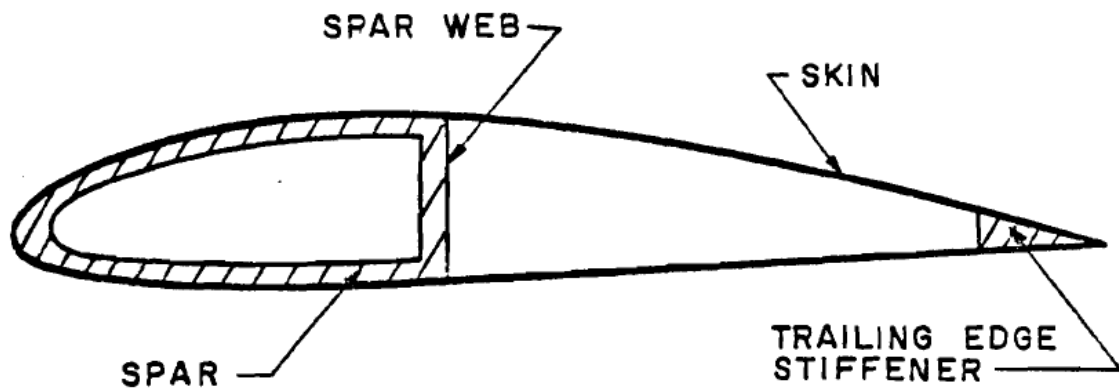


Figure 2. Cross section of windmill blade as per NACA 4415 airfoil.

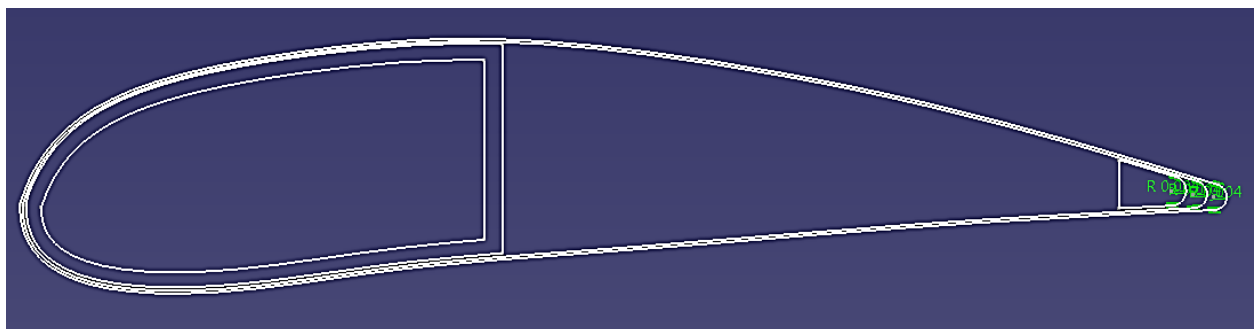


Figure 3. CATIA sketch of blade cross section.

Figure 3 shows distinctively the spar, web, stiffener and the skin of the blade. The corner of the blade is filleted to allow proper meshing of the model. The top view of the blade along with its design parameters are shown in Figure 4. as per [8].

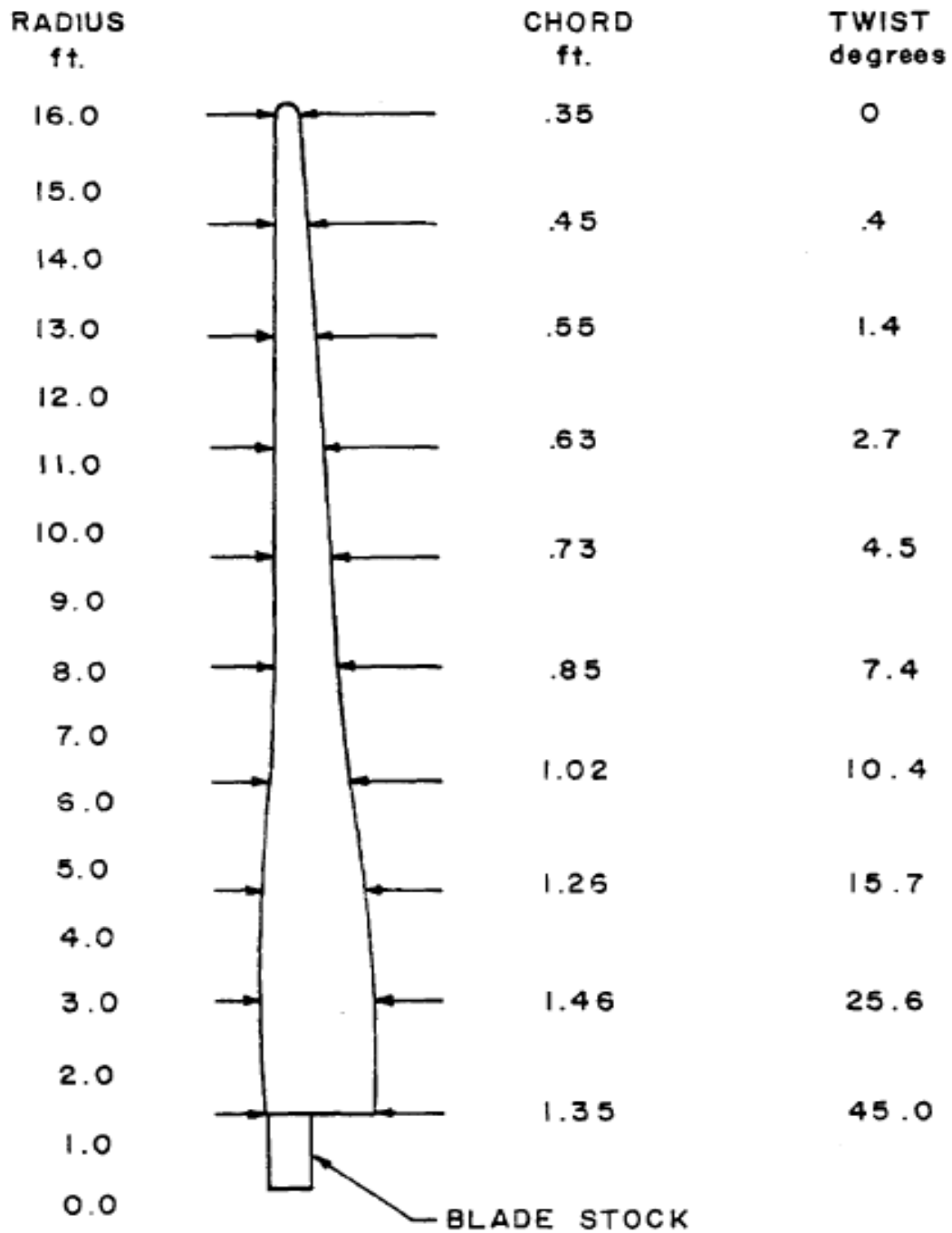
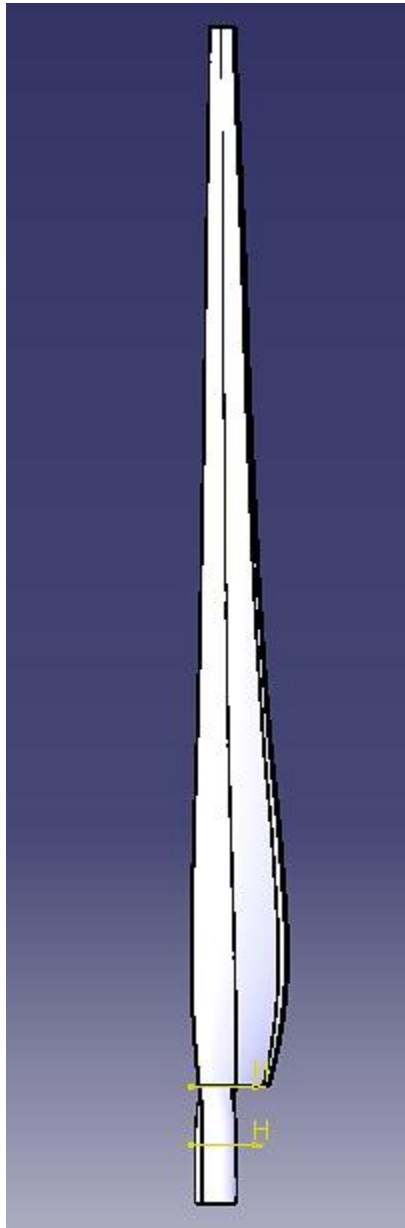


Figure 4. Blade geometry as per ref. [8]



**Figure 5. CATIA part corresponding to the above geometry.**

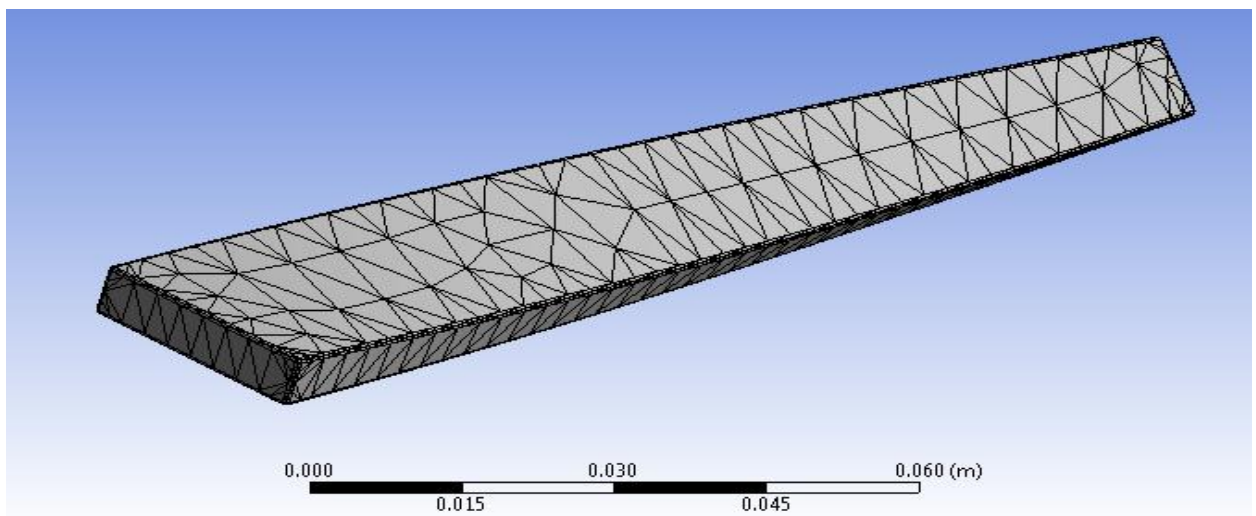
The model shown in Figure 5 was created by the method of Multi-Sections Solid. Planes were created along the blade span and cross sectional geometries were created on these planes as per [8]. Then these sections were joined by a flow of solid by the multi-sections solid method using ratio coupling mode. Extreme care was taken to create proper closing points on the sections to avoid forming of cusps during solid flow. The normal procedure is to model a part in CATIA and

then import it to ANSYS. But ANSYS takes up the geometry as a single body and hence separation of bodies in the blade becomes almost impossible if there are no clear dividing surfaces between the bodies. Therefore an innovative method was used to design the separate bodies by providing a clearance between them so that they do not block the separating surfaces. The advantage of doing this is that when we import this model to ANSYS, we can slice off the bodies to produce separate bodies by selecting the free faces. By separating the bodies, we can apply different material properties to them as per need.

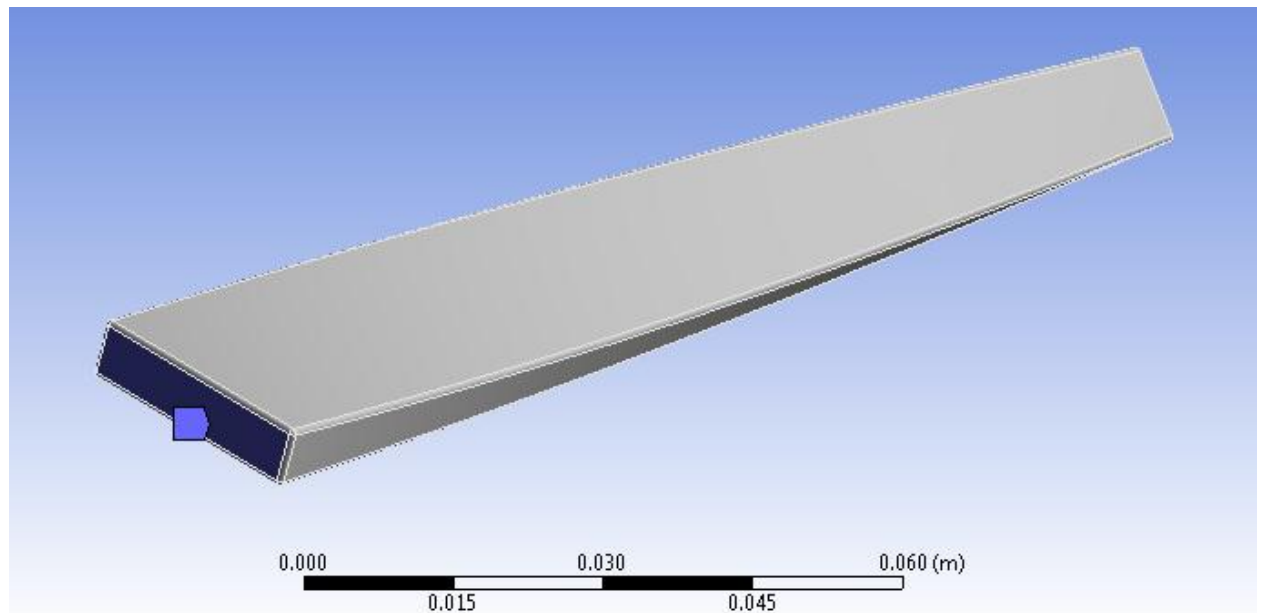
## ***5.2.MODAL ANALYSIS***

### **5.2.1. Modal analysis of Timoshenko beam as specified in [7]:**

Dynamic vibration analysis of the beam specified in [7] and modeled in CATIA was done in ANSYS workbench. The meshed beam looked as shown in Figure 6. This FEM model consisted of 6082 nodes and 3349 elements. The broader and thicker end, which is also the root of the beam, was given a fixed support as shown in Figure 7.



**Figure 6. Meshed model of beam as per ref. [7]**

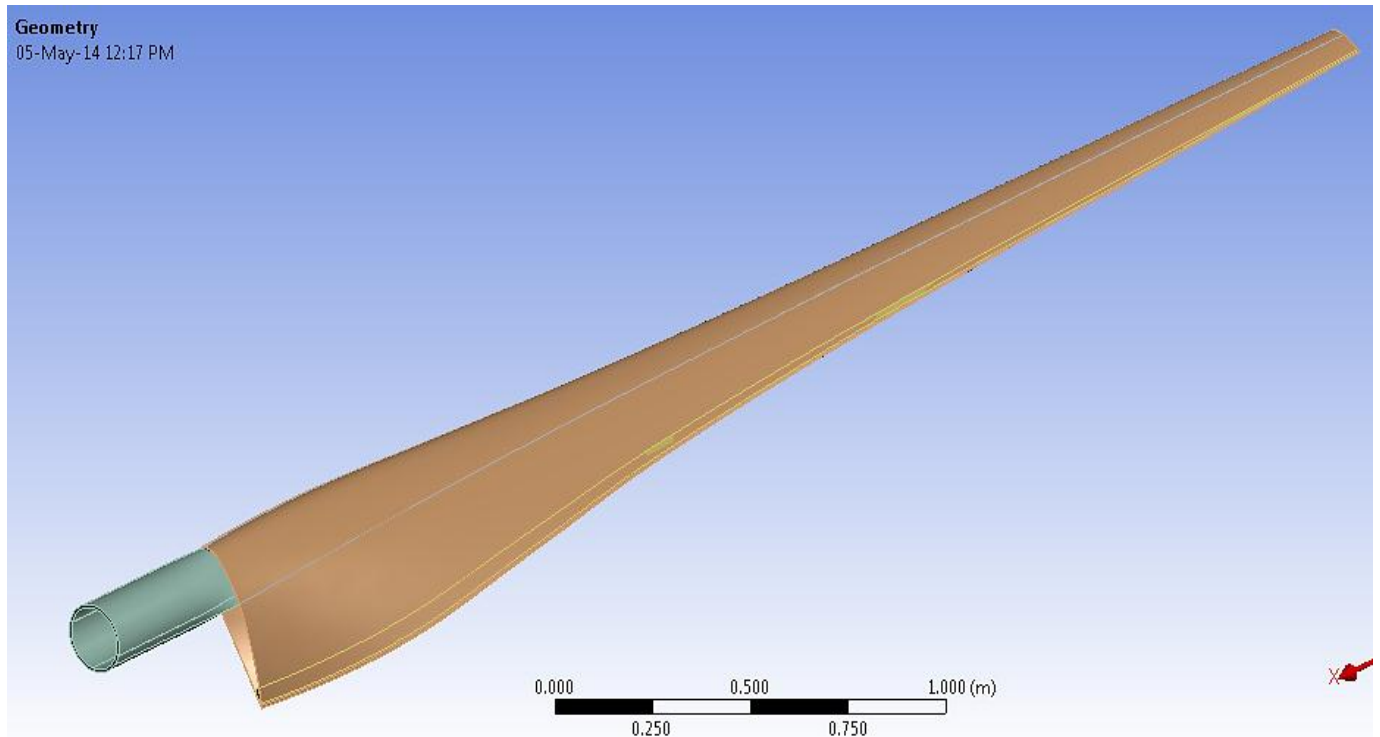


**Figure 7. Boundary condition for the beam.**

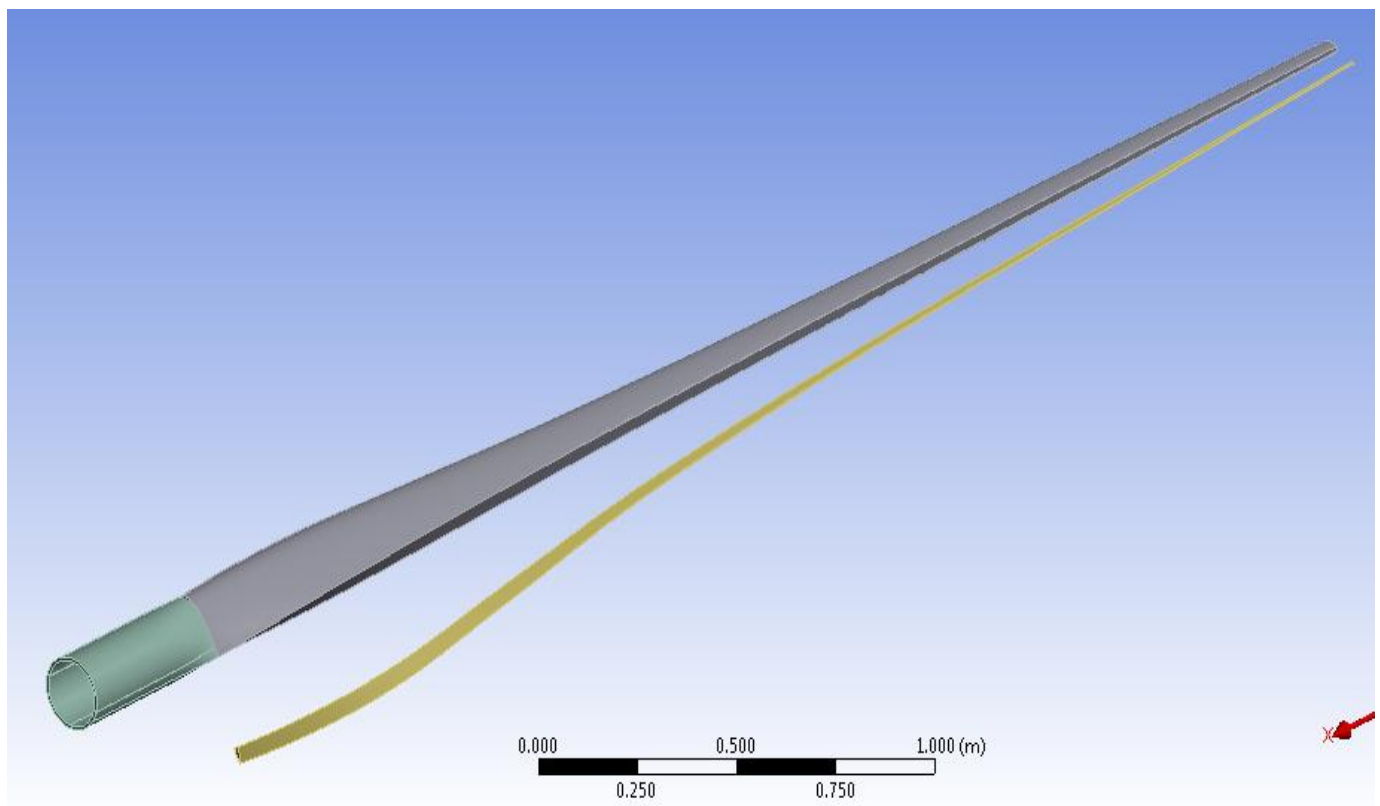
This boundary condition applied to beam is Cantilever type. The beam root is fixed rigidly with 0 DOF and the narrower end is the free end. The first 4 modes to be calculated are selected in the analysis settings option.

### **5.2.2. Modal Analysis of actual windmill blade model [8]:**

Modal analysis of the true blade model was done in ANSYS workbench. But before that, contact elements and properties had to be created as per the model requirements since we had allowed clearance separations between the bodies while creating the model. The model with and without the skin is shown in Figure 8 and Figure 9 respectively.



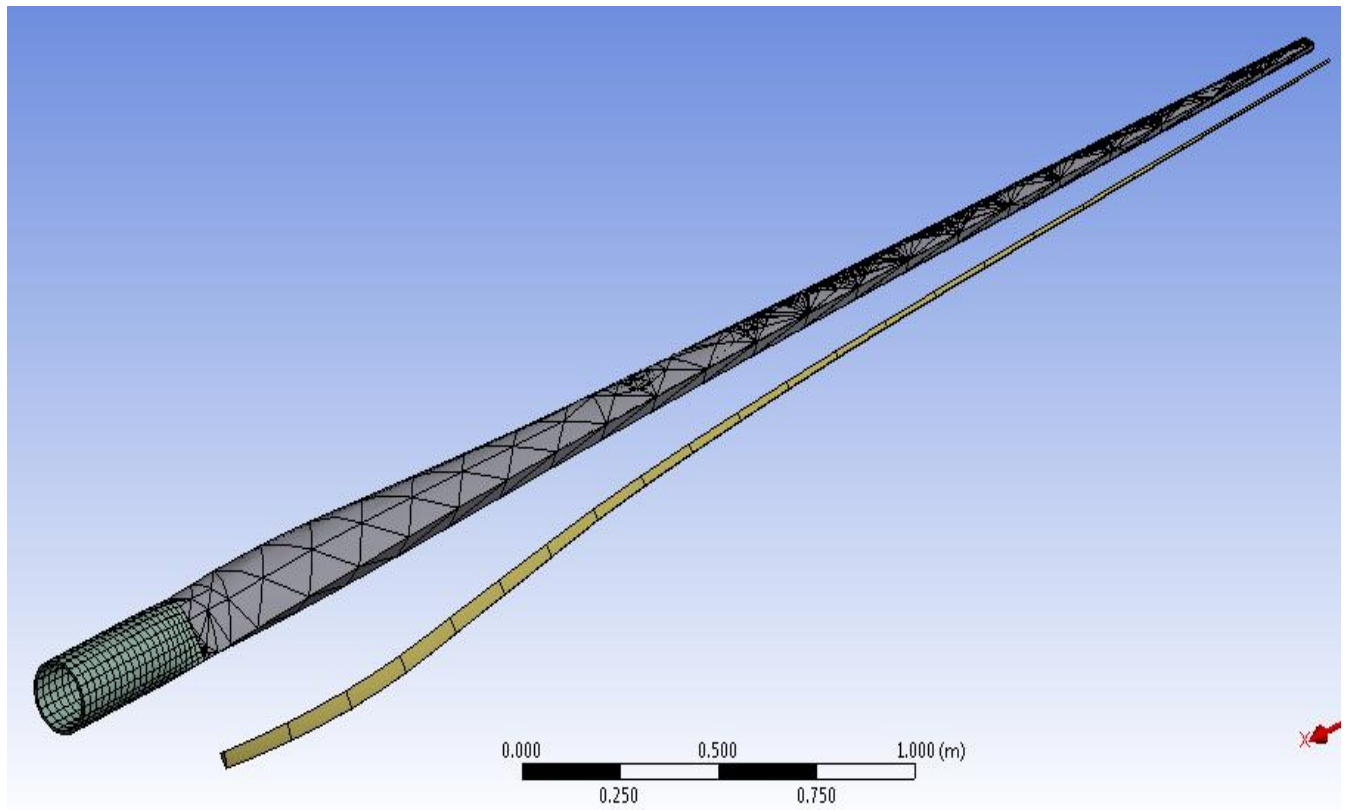
**Figure 8. Actual blade geometry after importing to ANSYS.**



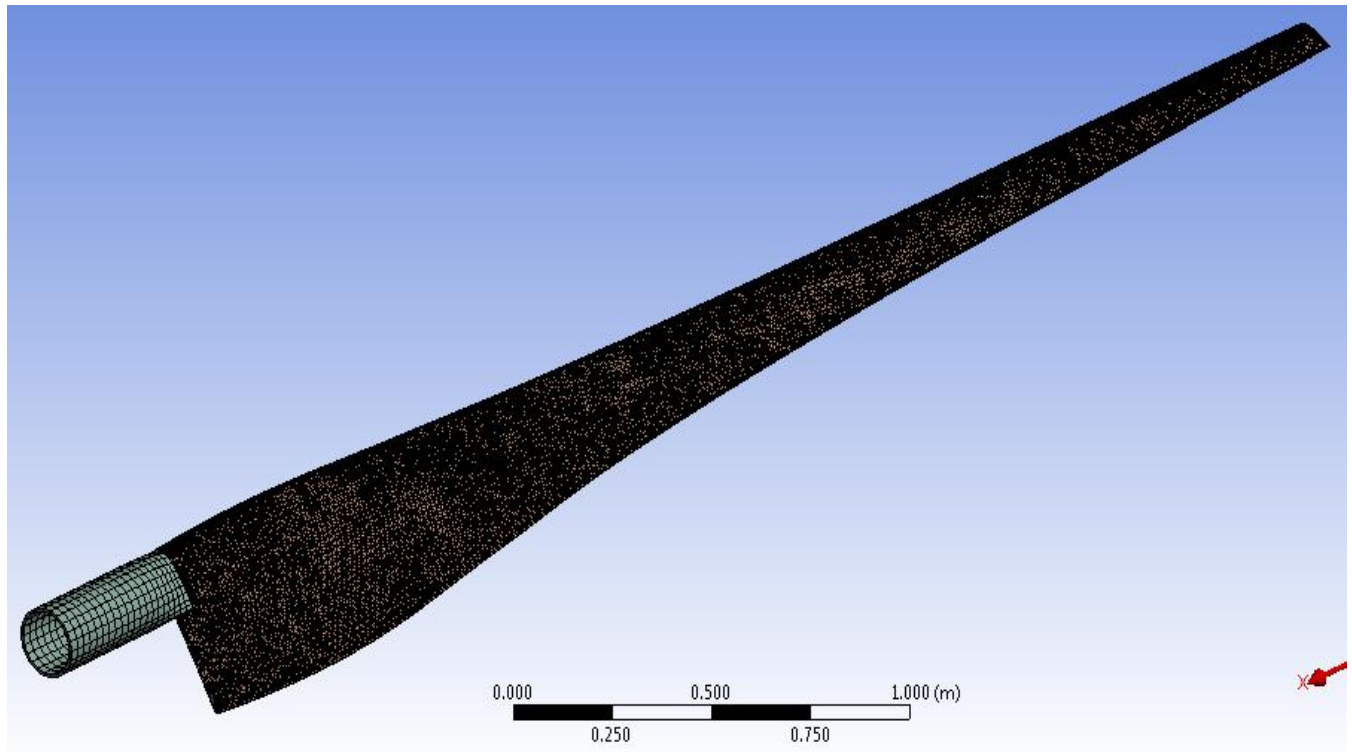
**Figure 9. Blade geometry with skin hidden to show inner parts.**



Figure 9 provides a detailed view of the various bodies that are present in the model as characterized by different colors. The meshed structure of the blade model without the skin and with skin is as shown in Figure 10 and Figure 11 respectively.



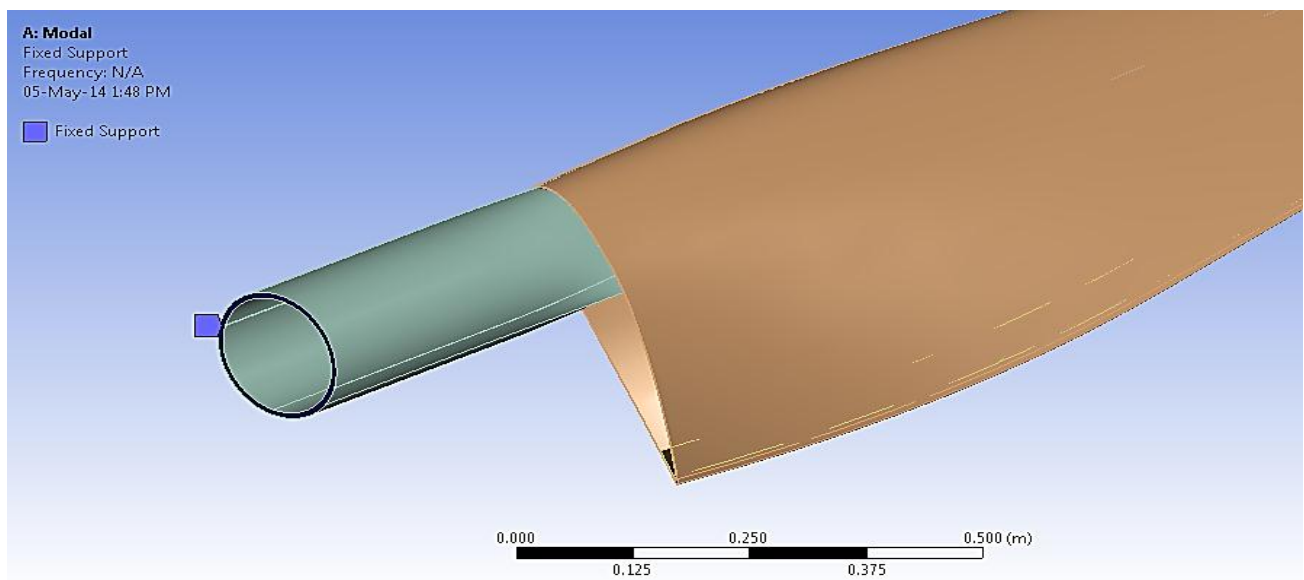
**Figure 10. Meshed interior of the blade.**



**Figure 11. Meshed view of the blade with skin.**

The boundary condition is same as that of a cantilever beam, i.e. one end fixed and one end free.

The root of the blade at the stock is fixed with 0 DOF as shown in Figure 12.

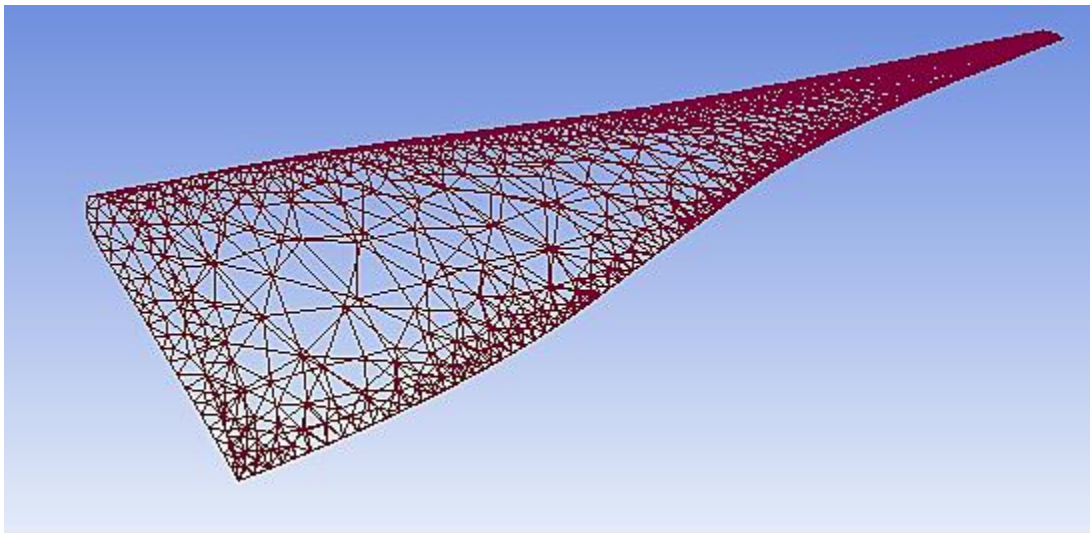


**Figure 12. Boundary condition for the blade model.**

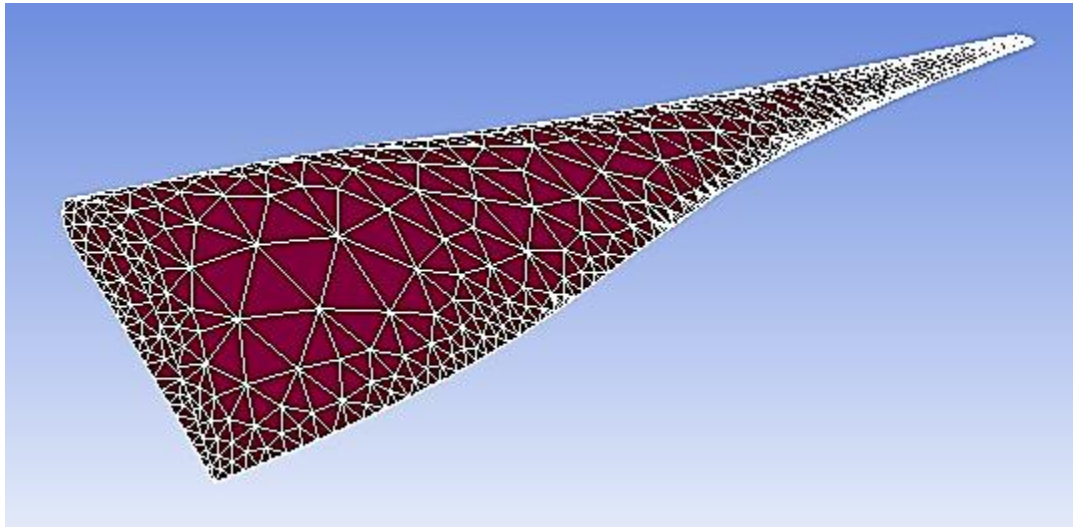
Three contact pairs had to be created using the bonded surface condition for skin-end stiffener, skin-spar and web, stock-spar and web pairs. A virtual topology consisting of all the outer faces of the skin had to be created for easier meshing of the skin part body. Analysis settings were set to compute and display 3 mode shapes so as to compare with the published research paper [8].

### ***5.3.CFD ANALYSIS***

The model of the realistic blade structure was imported to FLUENT for 3D CFD analysis. The finite volume method was used by the software to compute and display the wireframe meshed structure as shown in Figure 13 and surface frame mesh as shown in Figure 14.



**Figure 13. FEV model of blade in FLUENT as wireframe.**



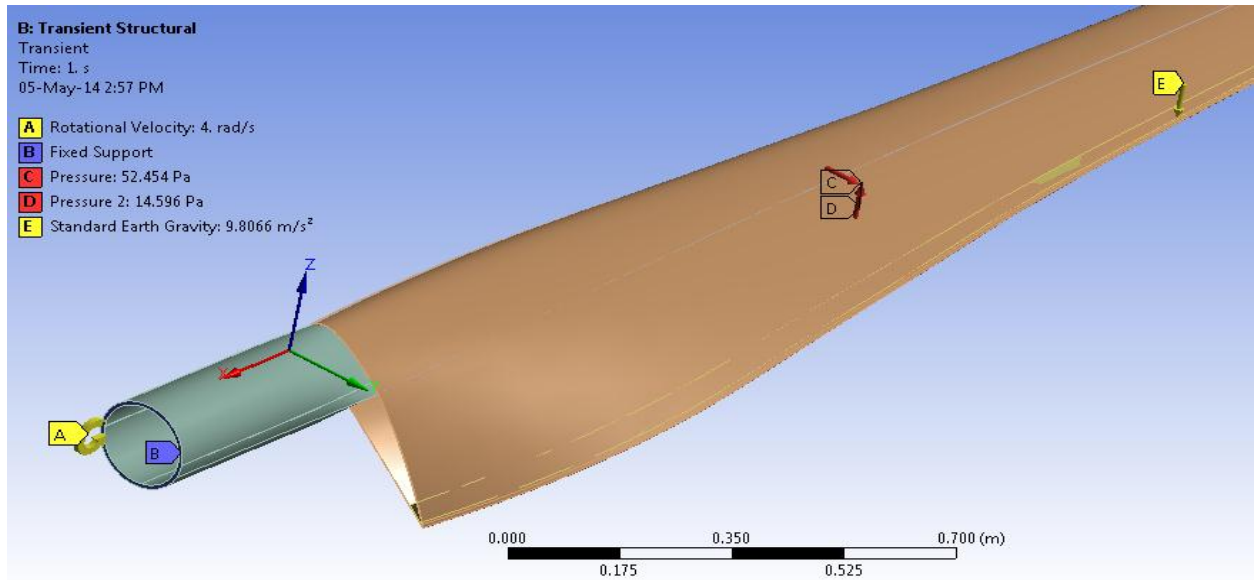
**Figure 14. FEV model of blade in FLUENT as surfaceframe.**

This finite volume model was put inside a bounding box so that external boundary flow analysis of the blade can be done. Then a Boolean operation was done to subtract the blade volume from the bounding box. Boundaries such as velocity inlet, pressure outlet, walls and blade wall were suitably selected to represent real world situations. The wind velocity given to the velocity inlet was 10m/s representing the maximum wind conditions on a windy day except for stormy weather when wind speeds can reach 60-75 m/s. The gauge pressure on the pressure outlet side was set to 0 Pascal.

#### ***5.4.STRESS ANALYSIS***

After obtaining the lift and drag force values along with the position of center of pressure from the CFD analysis, this information was used to carry out a stress analysis of the blade. These forces were converted into pressure values by dividing them with the respected projected area pertaining to lift and drag (also obtained from CFD analysis).

These forces were applied to the lower surface of the blade skin in their respective directions. Rotational velocity was imparted to the blade about the Y axis and self-weight was also imposed on the blade at its center of gravity as shown in Figure 15.



**Figure 15. Boundary condition of the blade for stress analysis.**

## ***5.5.FATIGUE LIFE ANALYSIS***

Fatigue analysis was carried out for the blade in the workbench module. The material data consisting of S-N curves for the given components were obtained from various sources and fed into the software's engineering material database. A reversal ratio of 0.4 was selected for the loading cycle. The typical load cycle setup for the given problem statement is as shown in Figure 16.

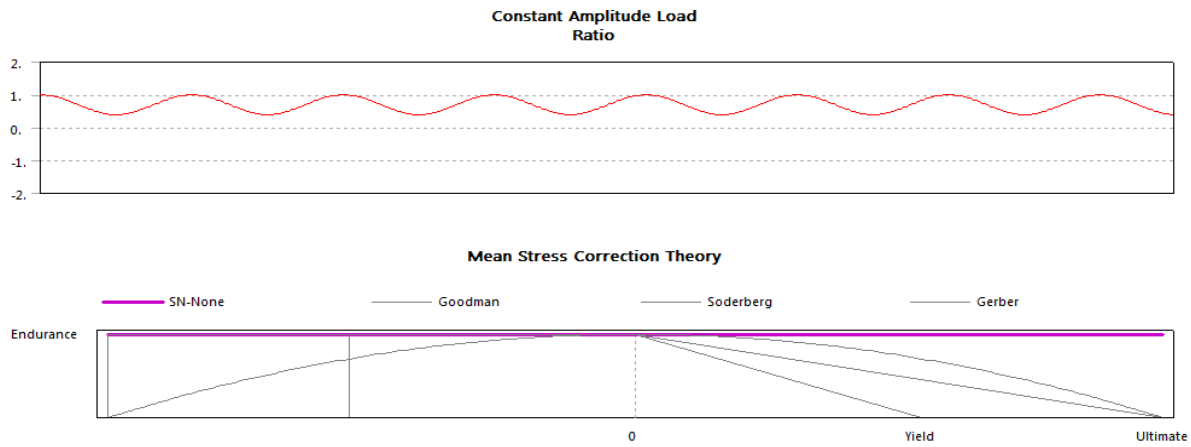


Figure 16. Fatigue load case.

## 6. MATERIALS USED

Two grades of steel were used for the beam and the blade differently, Epoxy-S-glass UD was used for the skin of the blade and Epoxy-Carbon UD 230GPa was used for the spar, web and stiffener. However for the validation of the natural frequencies, materials given in [8] were used for the vibration analysis. A new set of equivalently suitable materials were chosen because there was no definite property mentioned in [8] that could have assisted in carrying forth the fatigue analysis of the blade. The various material properties along with their S-N curves are given below.



i. **Materials as per [8]:** Materials properties used are given in Table 2.

Table 2. Material properties as specified in ref. [8]




Component	Density in $\text{kg/m}^3$	Young's modulus in GPa	Rigidity modulus in GPa
skin	1536.2347	15.168	3.447
spar	1386.763	30.336	2.068
stock	7850	199.947	76.945

**ii. Materials taken for final analysis:** Materials used for final analyses are given in Table 3-4 along with their properties. Epoxy-S-Glass UD is used as material for blade skin. Epoxy-carbon UD 230Gpa is used as material for spar, web and stiffener. Due to unavailability of material identities and relevant properties in [8], these equivalent materials were chosen suitably. Structural steel is used for sleeve.

**Table 3. Material property for Epoxy-S-glass UD material.**

Properties of Outline Row 7: Epoxy_SGlass_UD			
	A	B	C
1	Property	Value	Unit
2	 Density	2000	kg m <sup>-3</sup>
3	 Orthotropic Elasticity		
4	Young's Modulus X direction	50000	MPa
5	Young's Modulus Y direction	8000	MPa
6	Young's Modulus Z direction	8000	MPa
7	Poisson's Ratio XY	0.3	
8	Poisson's Ratio YZ	0.4	
9	Poisson's Ratio XZ	0.3	
10	Shear Modulus XY	5000	MPa
11	Shear Modulus YZ	3846.2	MPa
12	Shear Modulus XZ	5000	MPa

**Table 4. Material properties for Epoxy-carbon UD 230Gpa.**

Properties of Outline Row 5: Epoxy_Carbon_UD_230GPa_Pregreg			
	A	B	C
1	Property	Value	Unit
2	 Density	1490	kg m <sup>-3</sup>
3	 Orthotropic Secant Coefficient of Thermal Expansion		
9	 Orthotropic Elasticity		
10	Young's Modulus X direction	1.21E+05	MPa
11	Young's Modulus Y direction	8600	MPa
12	Young's Modulus Z direction	8600	MPa
13	Poisson's Ratio XY	0.27	
14	Poisson's Ratio YZ	0.4	
15	Poisson's Ratio XZ	0.27	
16	Shear Modulus XY	4700	MPa
17	Shear Modulus YZ	3100	MPa
18	Shear Modulus XZ	4700	MPa

Figures 17, 18, 19 depict the S-N curves of selected materials used for different blade part models. Figure 20 gives an S-N plot for fiber materials as taken from ref. [9].

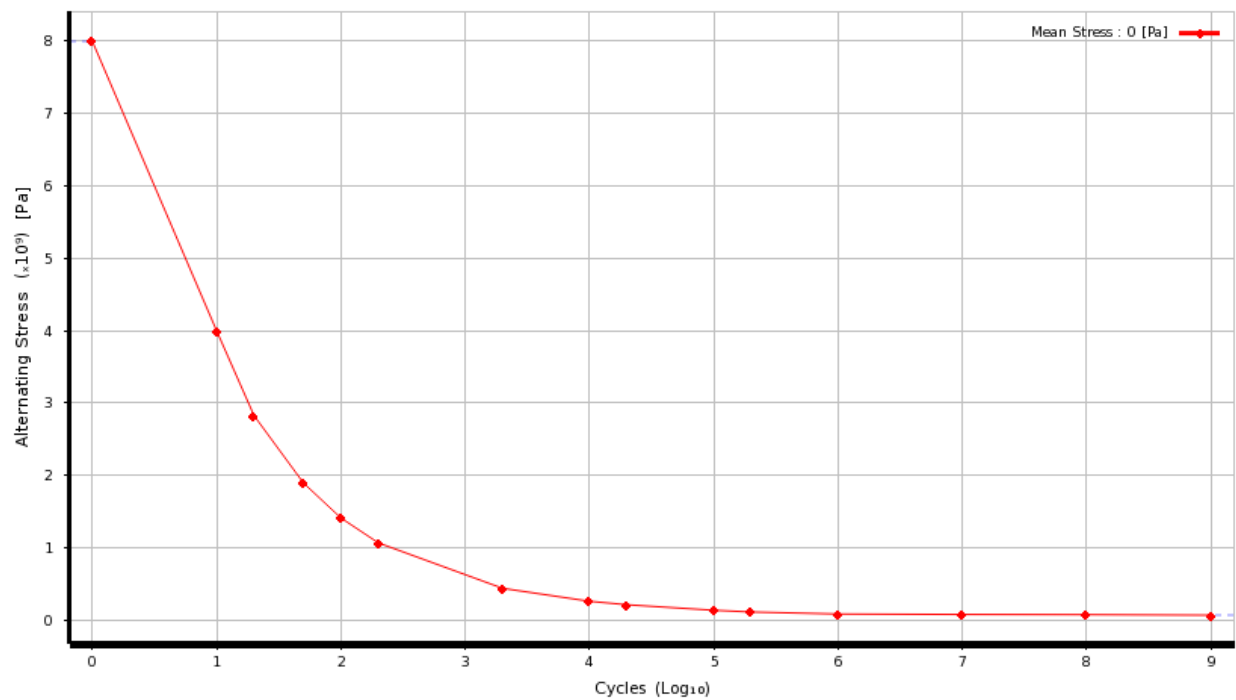


Figure 17. S-N curve of structural steel.

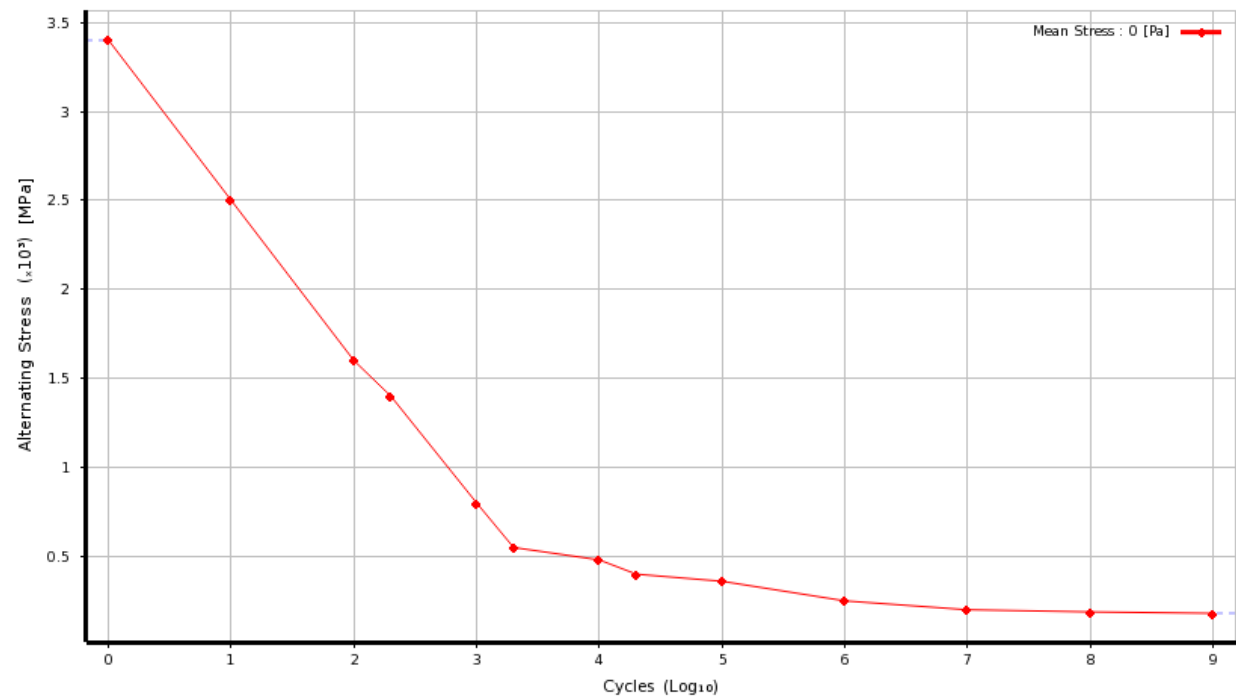


Figure 18. S-N curve for epoxy-S-glass.



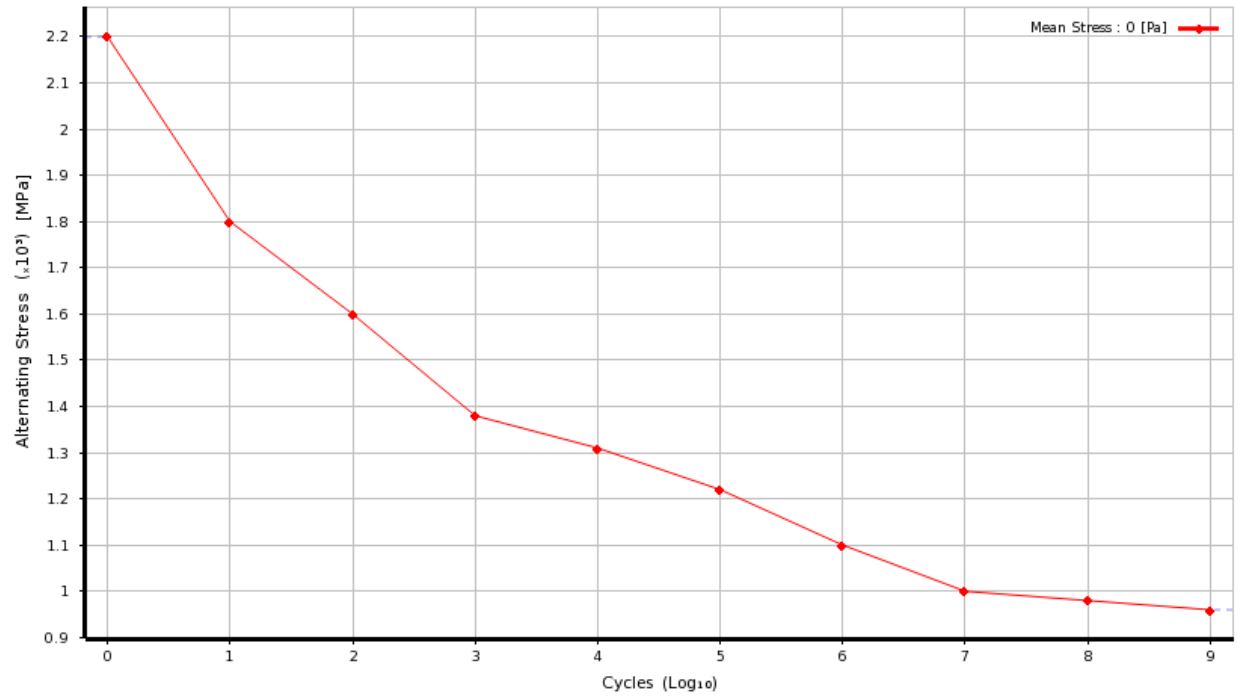


Figure 19. S-N curve for Epoxy-carbon UD 230Gpa.

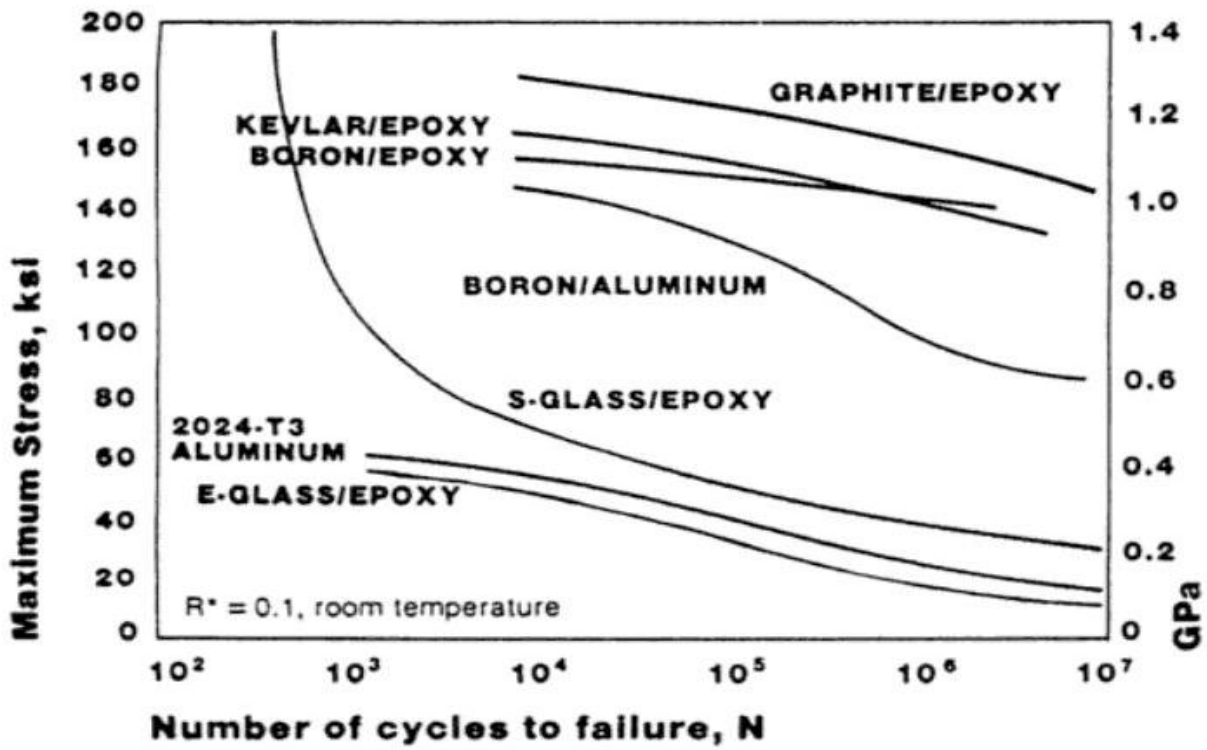


Figure 20. Reference curves for various useful materials [9].

## 7. BLADE GEOMETRIC PARAMETERS

The geometric properties of the windmill blade are given in Tables 5 and 6. They give information about geometric and physical attributes of the solid model.

**Table 5. Geometric properties of Blade.**

Bounding Box	
Length X	4.8768 m
Length Y	0.40435 m
Length Z	0.32818 m
Properties	
Volume	8.5411e-003 m <sup>3</sup>
Mass	24.49 kg
Scale Factor Value	1.
Statistics	
Bodies	4
Active Bodies	4
Nodes	209301
Elements	104205

**Table 6. Blade components geometric and physical parameters.**

Material				
Assignment	Epoxy_SGlass_UD	Structural Steel	Epoxy_Carbon_UD_230GPa_Pregreg	
Nonlinear Effects	Yes			
Thermal Strain Effects	Yes			
Bounding Box				
Length X	4.3891 m	0.48768 m	4.3891 m	
Length Y	0.40435 m	0.16781 m	0.21854 m	0.18387 m
Length Z	0.29583 m	0.16773 m	0.20844 m	0.12871 m
Properties				
Volume	1.6584e-003 m³	1.7303e-003 m³	7.1207e-004 m³	4.4403e-003 m³
Mass	3.3167 kg	13.496 kg	1.061 kg	6.6161 kg
Centroid X	-1.17 m	0.25785 m	-1.2269 m	-1.2715 m
Centroid Y	0.17446 m	0.10413 m	0.31899 m	9.7015e-002 m
Centroid Z	-1.663e-002 m	3.045e-002 m	-7.4204e-002 m	1.5622e-002 m
Moment of Inertia Ip1	3.822e-002 kg·m²	8.0744e-002 kg·m²	1.2179e-003 kg·m²	1.8097e-002 kg·m²
Moment of Inertia Ip2	3.3911 kg·m²	0.28409 kg·m²	1.0278 kg·m²	6.9753 kg·m²
Moment of Inertia Ip3	3.4224 kg·m²	0.28781 kg·m²	1.0272 kg·m²	6.985 kg·m²
Statistics				
Nodes	193684	3917	272	11428
Elements	97703	660	22	5820

## 8. RESULTS AND DISCUSSIONS

### 8.1. MODAL ANALYSIS OF BEAM

The modal analysis of the Timoshenko beam in ANSYS resulted in giving the values of the first 4 natural frequencies of the beam. These values were compared with those obtained from the reference [7]. The mode shapes of the beam are shown in Figures 21-24 in order of their harmonics. A comparison Table of natural frequency values from present work and from [7] is given in Table no. 7.

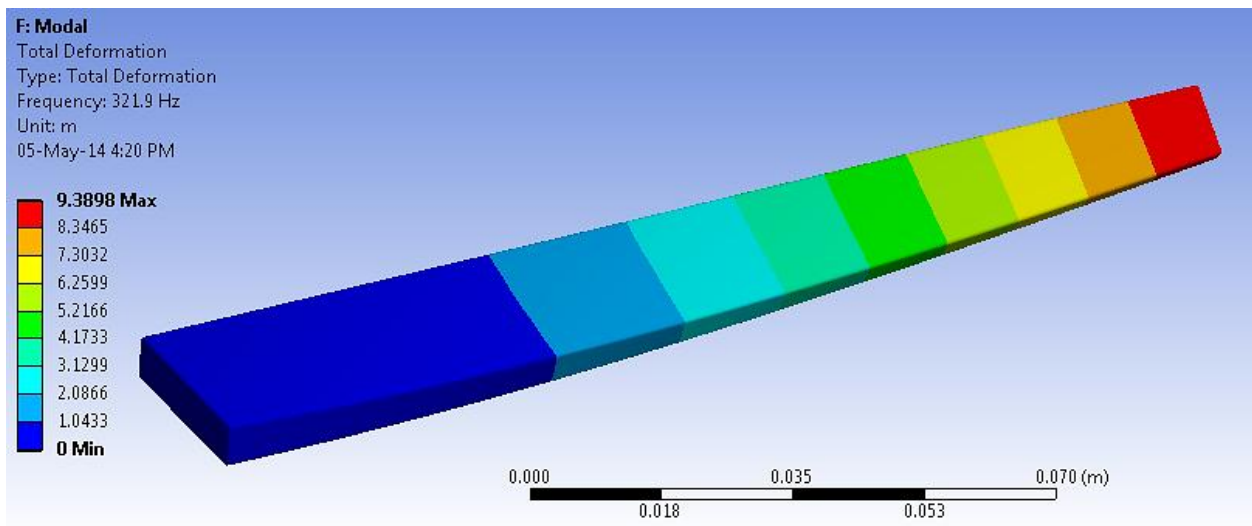
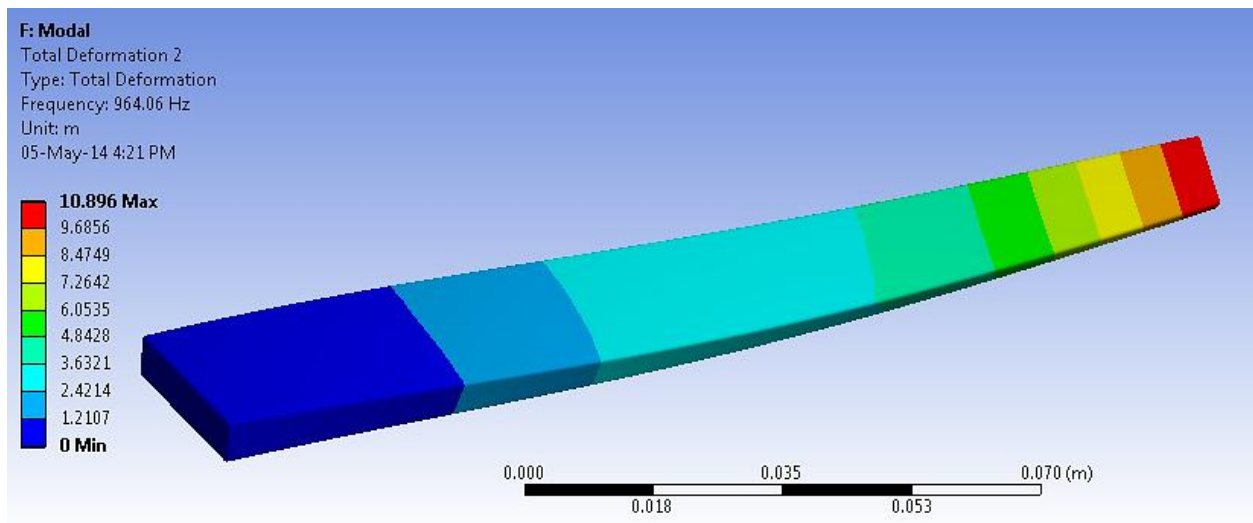
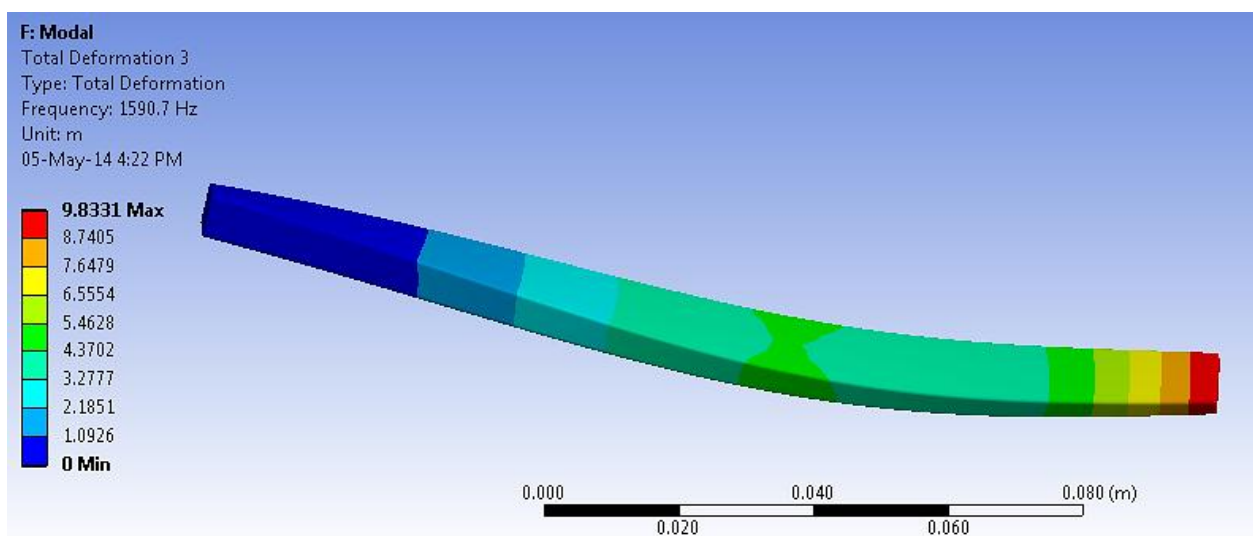


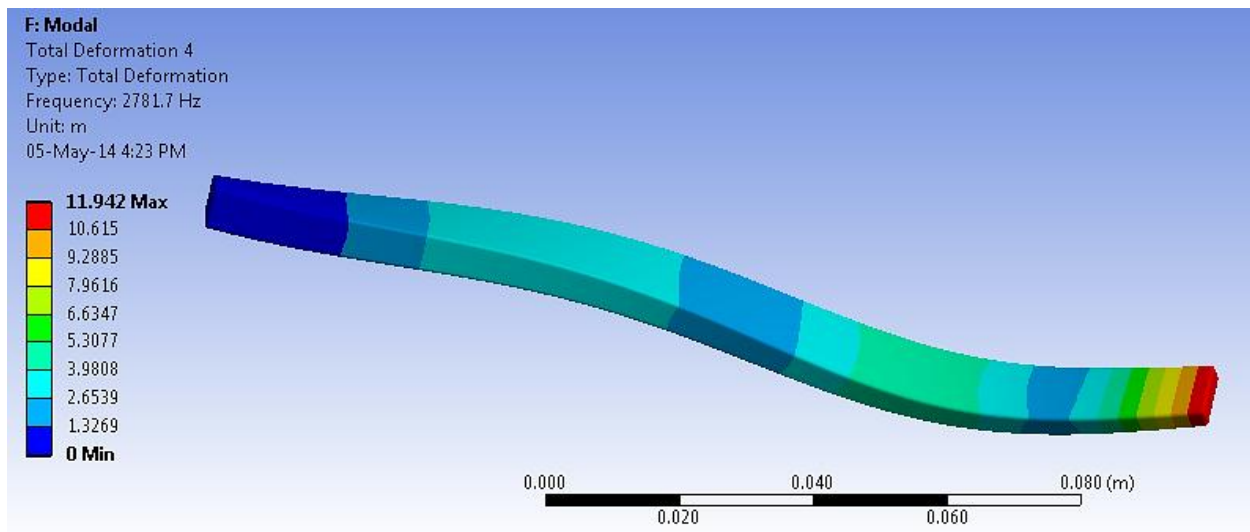
Figure 21. Mode shape of blade corresponding to 1st modal frequency.



**Figure 22. Mode shape of blade corresponding to 2nd modal frequency.**



**Figure 23. Mode shape of blade corresponding to 3rd modal frequency.**



**Figure 24. Mode shape of blade corresponding to 4th modal frequency.**

**Table 7. Comparison of beam's natural frequencies.**

Mode Shape	Natural frequency from ANSYS	Natural frequency ref. [7]
1	321.9 Hz	297.8 Hz
2	964.06 Hz	1137.3 Hz
3	1590.7 Hz	1645 Hz
4	2781.7 Hz	3578.3 Hz

From the above Table we can infer that obtained primary natural frequency is higher than its corresponding ref. [7] value. The subsequent frequency values are lower than their counterparts in ref. [7]. This is a desirable trend.

## 8.2. MODAL ANALYSIS OF TRUE BLADE MODEL

The true blade model as taken from [8] was passed through a modal analysis in ANSYS workbench and the results were compared with that of the reference research paper [8]. The first 3 natural vibrational frequencies are obtained. The corresponding mode shapes are shown in Figures 25-27. Comparison of frequency values from both ANSYS and [8] are tabulated in Table 8.

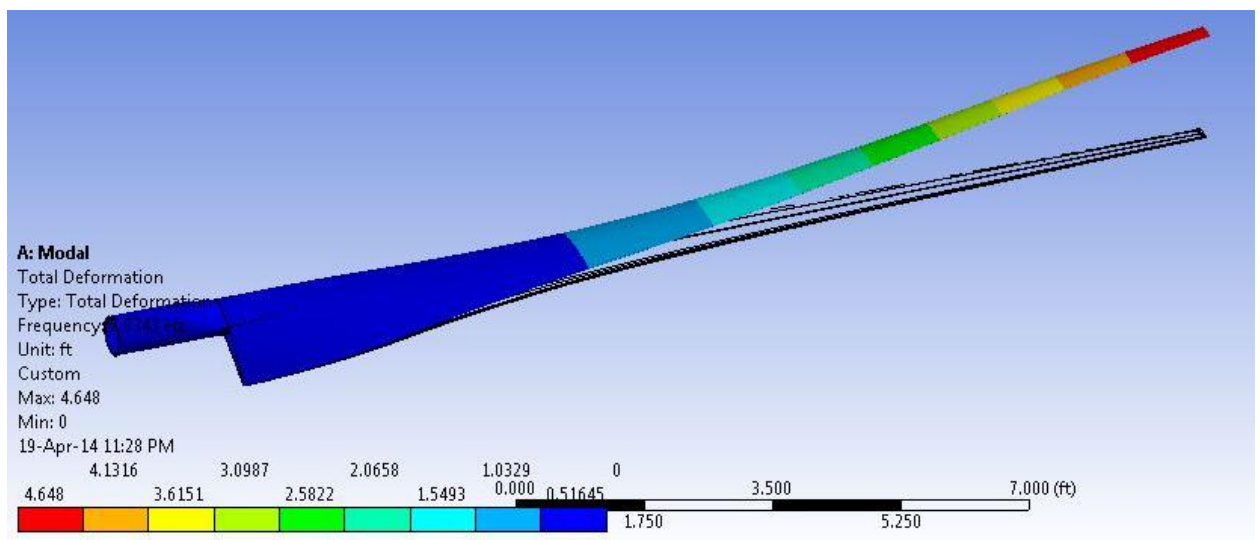


Figure 25. 1st mode shape of blade.

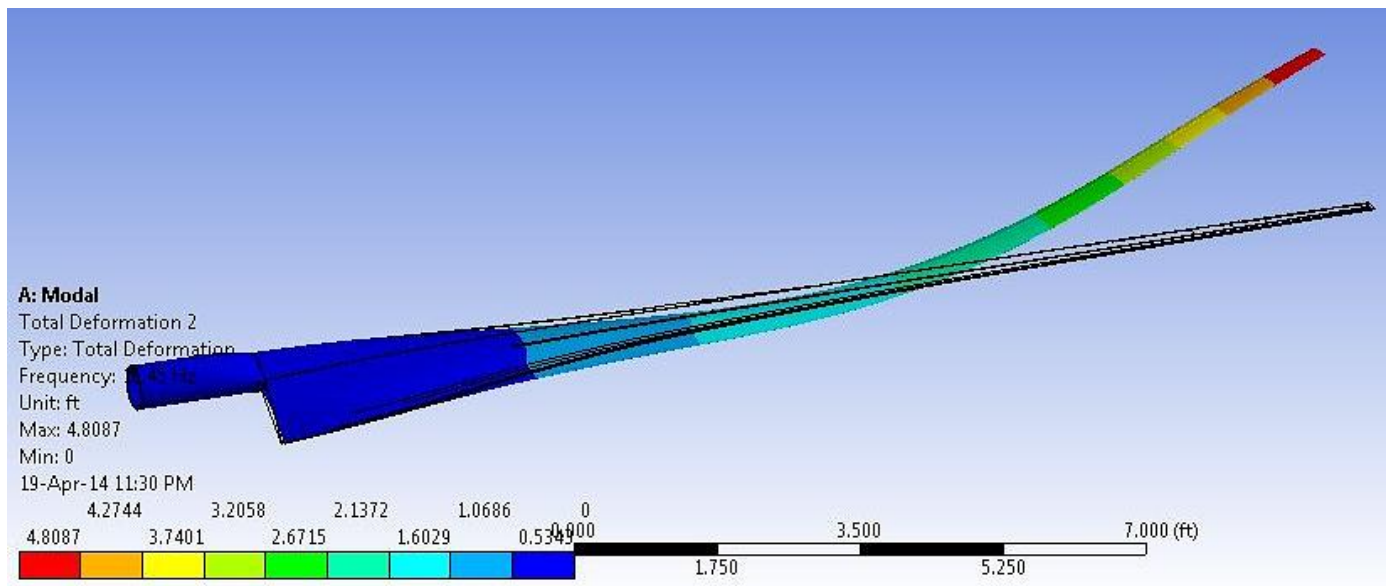


Figure 26. 2nd mode shape of blade.

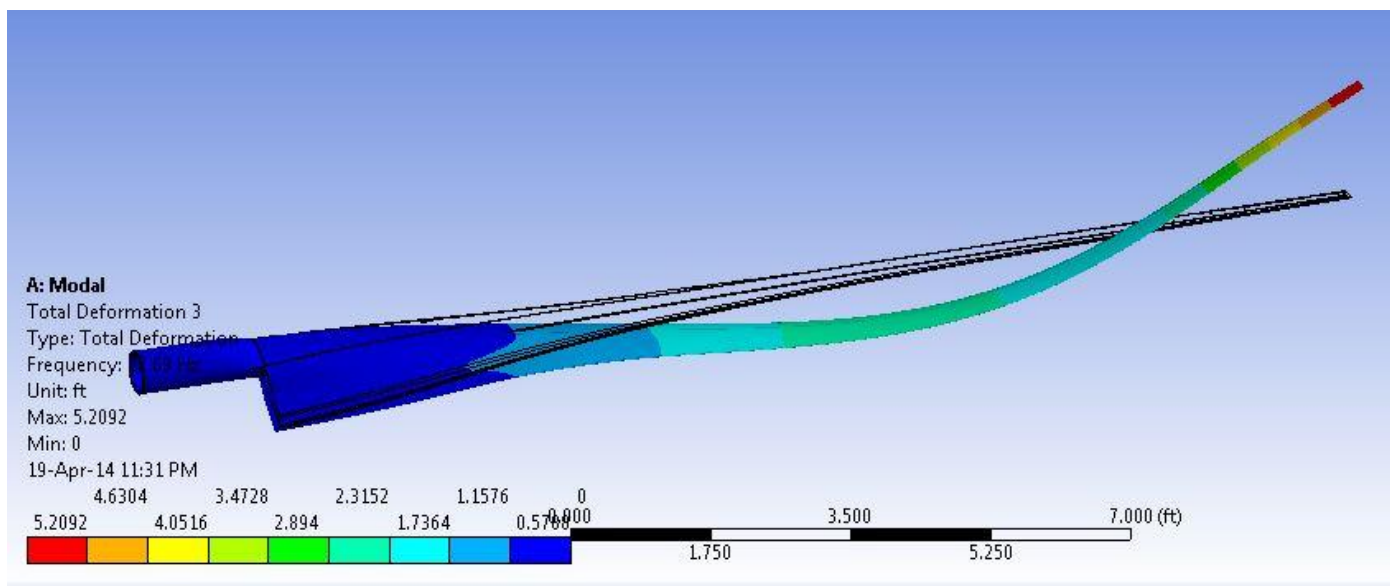


Figure 27. 3rd mode shape of blade.

**Table 8. Comparison of blade's modal frequencies.**

Mode Shape	Obtained natural frequency in Hz	Predicted natural frequency ref. [8] in Hz	Measured natural frequency ref. [8] in Hz
1	5.8751	4.4585	3.98
2	9.9044	10.35	8.917
3	15.025	14.809	13.375

From the analysis of the above Table we can infer that the values of calculated natural frequencies and observed natural frequencies are quite in good agreement with each other. Thus the blade model is validated with the existing published research paper.

### ***8.3.CFD ANALYSIS OF TRUE BLADE MODEL IN FLUENT***

Computational Fluid Dynamics analysis of the blade model was done in ANSYS FLUENT module. The pressure contours and velocity vector field around the skin of the blade were observed and analyzed.



8.3.1. Pressure contours

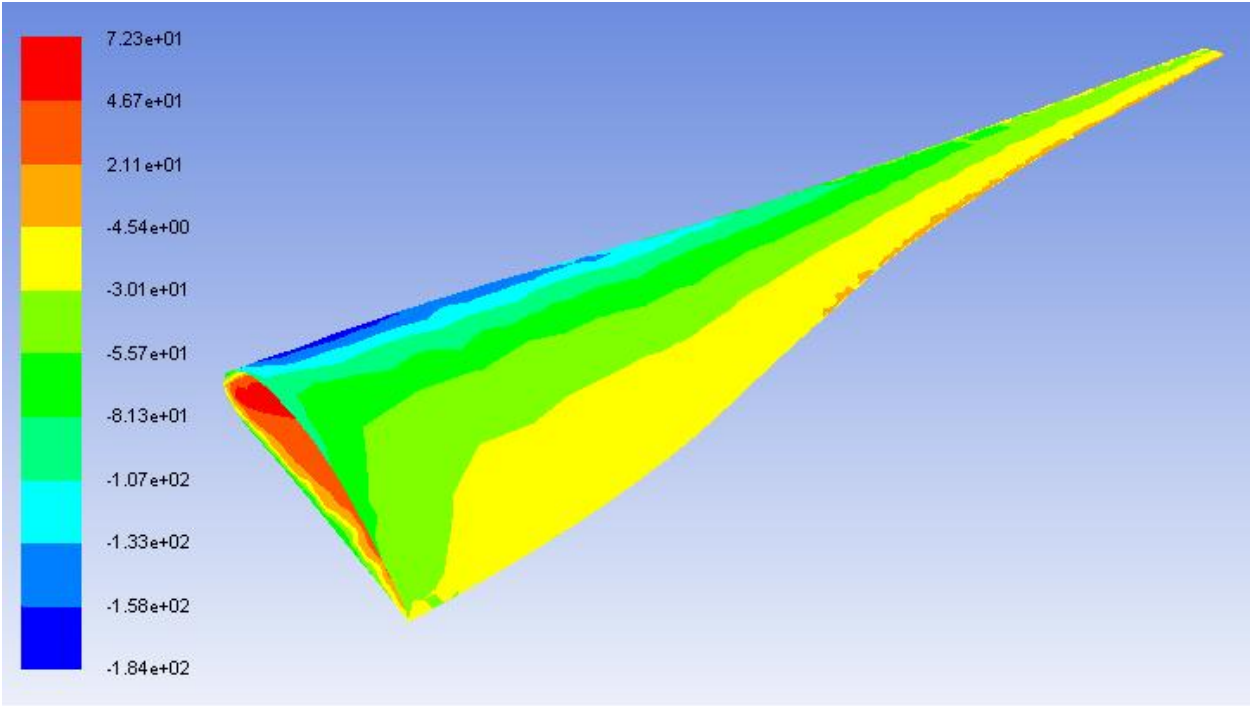


Figure 28. Pressure contour on upper skin of blade.

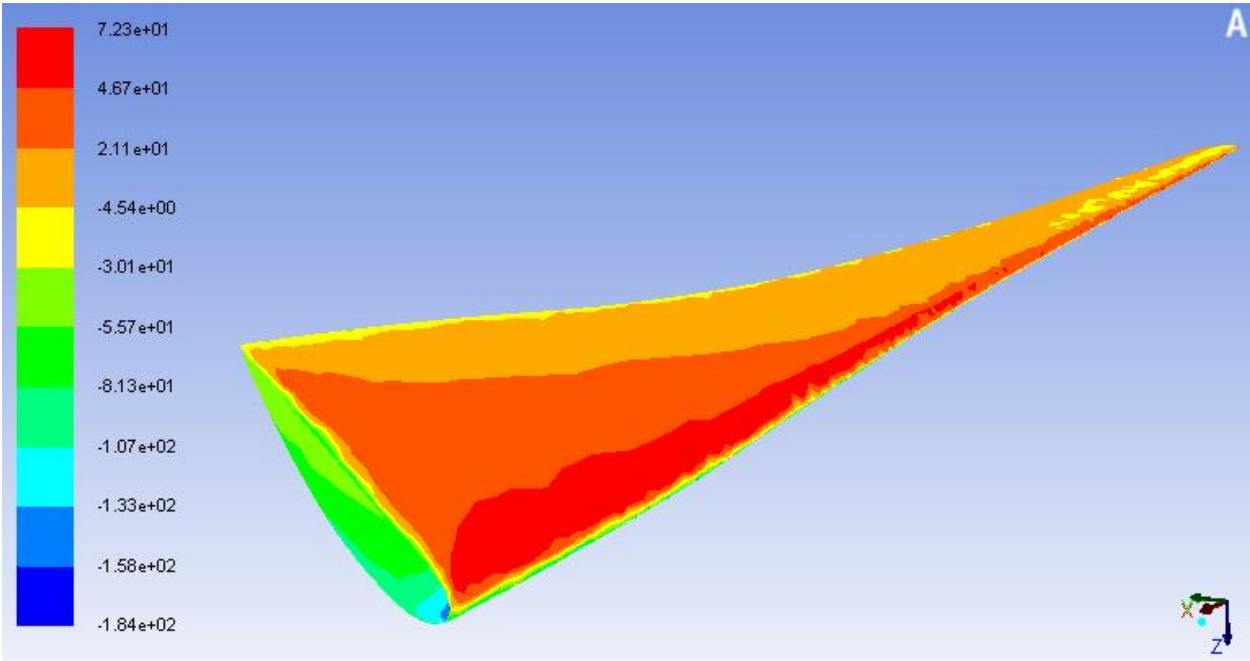
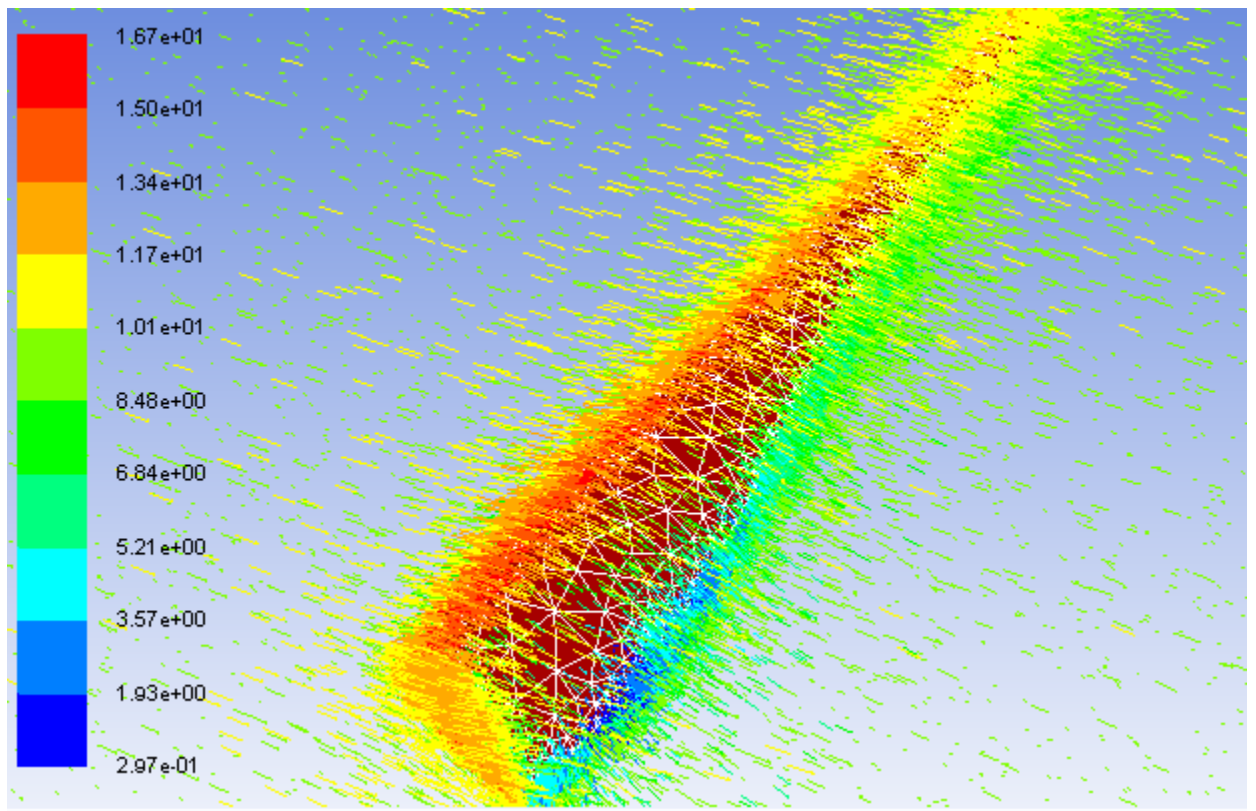


Figure 29. Pressure contour on lower skin of blade.

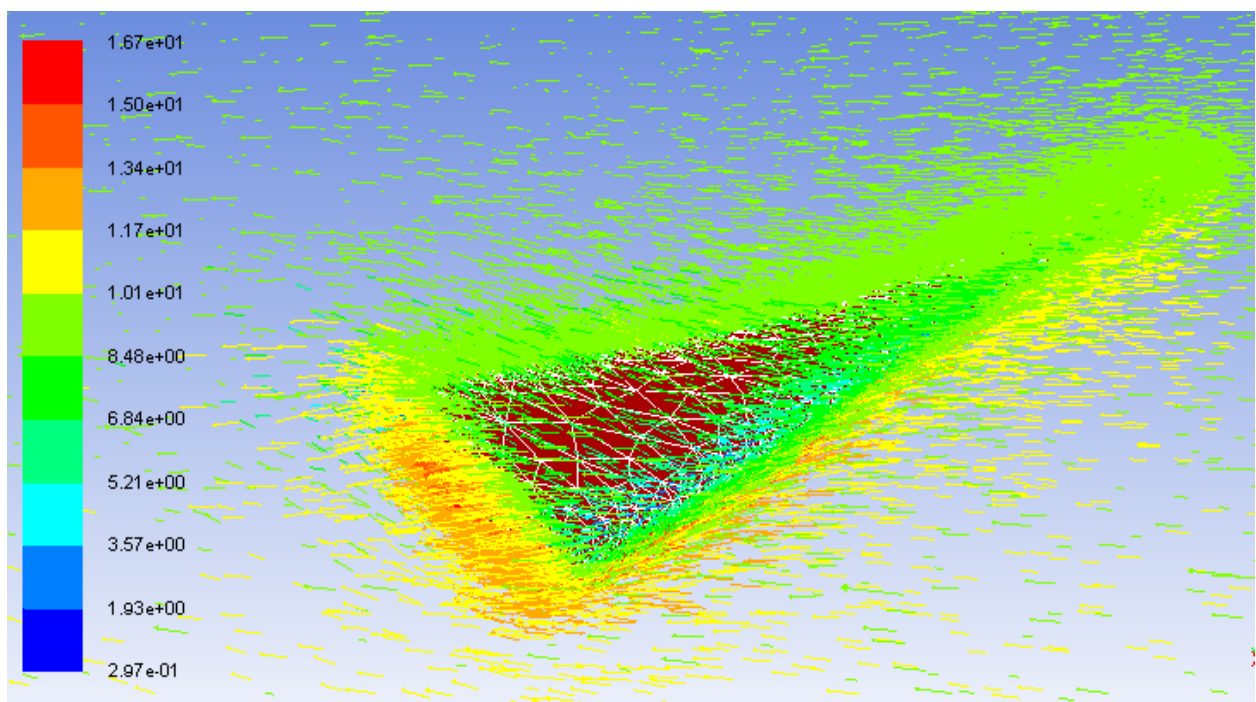
Figure 28 shows the distribution of static pressure over the upper skin of blade surface. It can be observed that the pressure values are on the lower side of the spectrum. Similarly, Figure 29 shows distribution of static pressure on the lower surface of blade skin. Observation shows that pressure on the lower side is relatively higher than pressure on upper side, as expected. This creates a net upward force which produces a lift.

### **8.3.2. Velocity vector field**

Figure 30 shows velocity vector field over the upper skin surface of the blade. It can be observed that the values of velocity are quite high and lie towards the red side of the spectrum. Figure 31 shows velocity vector field below the lower surface of blade skin. Observations reveal that these values are much lower than those corresponding to upper skin surface. Thus, by Bernoulli's principle, pressure below skin is greater than pressure above skin thereby leading to lifting the airfoil (blade) [15-17].



**Figure 30. Velocity vector field on upper skin of blade.**



**Figure 31. Velocity vector field on lower skin of blade.**

Table 9 gives the co-ordinates of the center of pressure of the blade in the air flow field.

Table 10 gives values of net drag force in the direction cosine (0 1 0) which is the direction parallel to wind flow. Table 11 gives values of net lift force in the direction cosine (0 0 1) which is the direction perpendicular to wind flow.

**Table 9. Listing of center of pressure for the blade model.**

<b>Center of Pressure - Set Coordinate z = 0 (m)</b>		
<b>Zone</b>	<b>x</b>	<b>y</b>
<b>skin</b>	<b>-1.2349878</b>	<b>0.1371638</b>
<hr/>		
<b>Net</b>	<b>-1.2349878</b>	<b>0.1371638</b>

**Table 70. Listing of drag force for the blade model.**

<b>Forces - Direction Vector (0 1 0)</b>			
	<b>Forces (n)</b>		
<b>Zone</b>	<b>Pressure</b>	<b>Viscous</b>	<b>Total</b>
<b>skin</b>	<b>18.802754</b>	<b>1.1579437</b>	<b>19.960697</b>
<hr/>			
<b>Net</b>	<b>18.802754</b>	<b>1.1579437</b>	<b>19.960697</b>

<b>Forces - Direction Vector (0 0 1)</b>			
	<b>Forces (n)</b>		
<b>Zone</b>	<b>Pressure</b>	<b>Viscous</b>	<b>Total</b>
<b>skin</b>	<b>69.123446</b>	<b>-0.12659076</b>	<b>68.996855</b>
<hr/>			
<b>Net</b>	<b>69.123446</b>	<b>-0.12659076</b>	<b>68.996855</b>

**Table 81. Listing of lift force values for the blade model.**

## 8.4. STRESS ANALYSIS

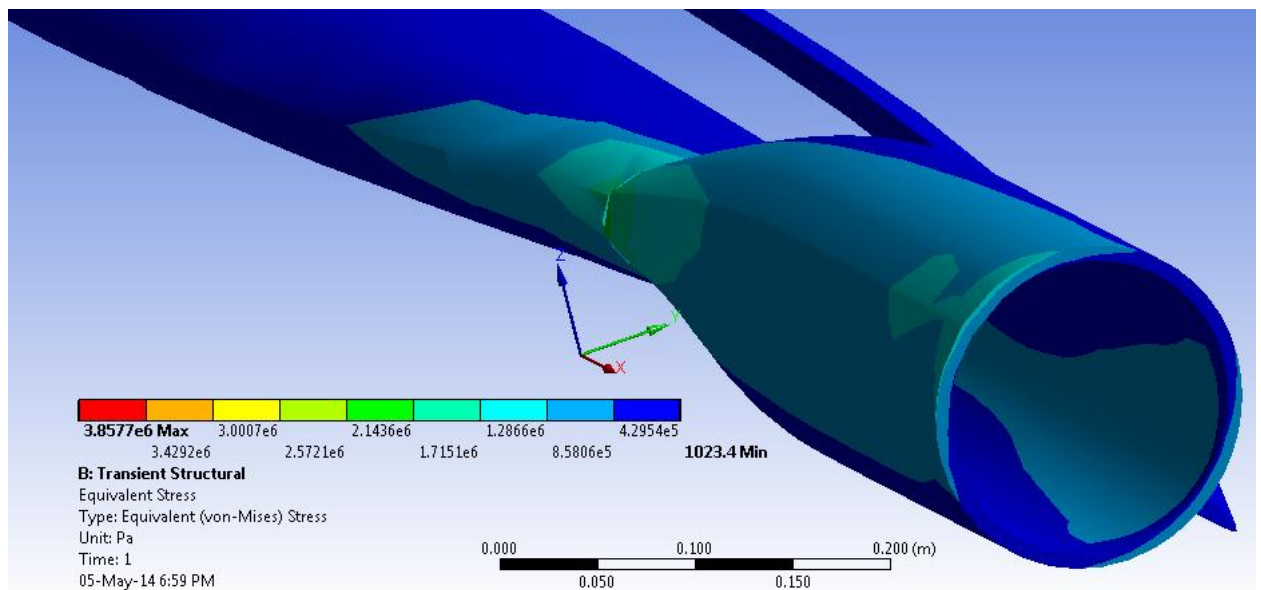


Figure 32. Hotspots of the blade geometry.

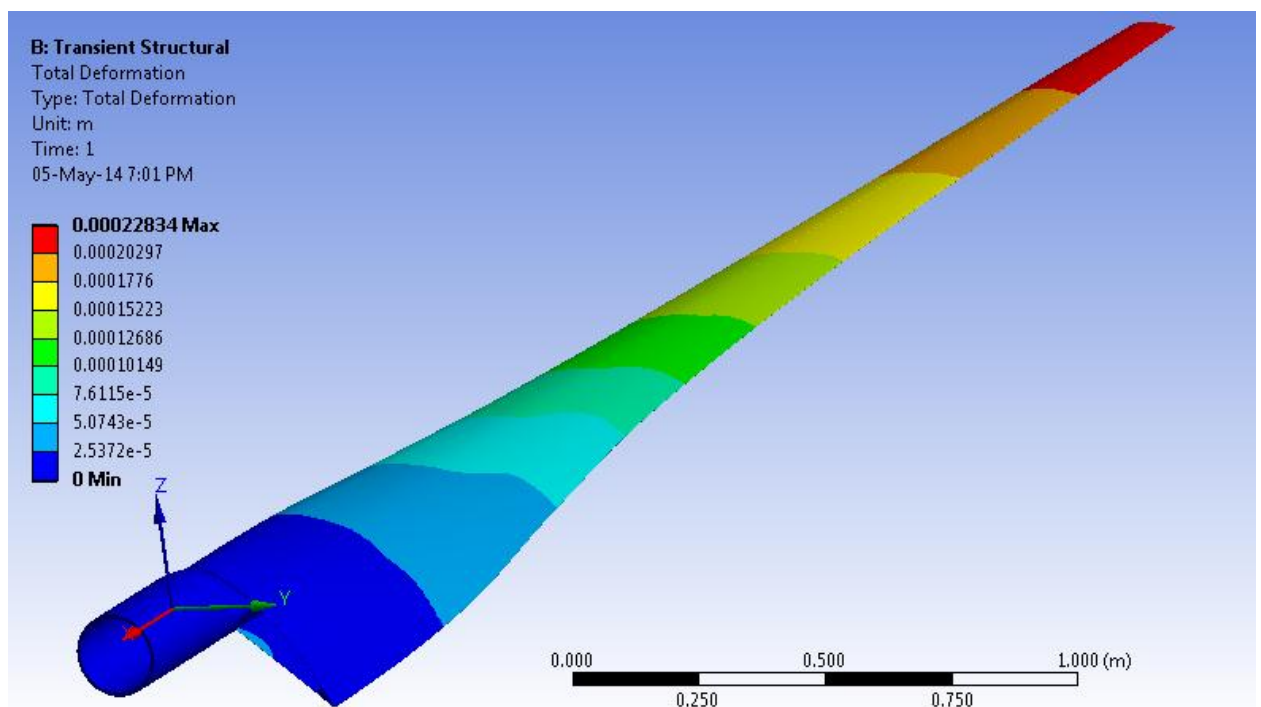


Figure 33. Deformation analysis of the windmill blade.

The stress analysis of the part model showed that maximum stresses are generated around the root of the blade and at the joining point of blade to stock as seen in Figure 32. Figure 33 shows the deformation contour along the blade span due to the applied loads. As evident, the tip of the blade has a maximum displacement of 0.228 mm.

## 8.5.FATIGUE LIFE PREDICTION

Table 12 enlists the results from the fatigue analysis of the actual blade model. It gives results in terms of number of cycles, factor of safety, design life and equivalent alternating stress.

**Table 92. Fatigue life prediction analysis of the blade model.**

Type	Life	Damage	Safety Factor	Equivalent Alternating Stress
Minimum	1.57e+009 seconds		> 15	307.02 Pa
Minimum Occurs On	skin		skin	
Maximum		1.		1.1573e+006 Pa
Maximum Occurs On		skin		skin

This fatigue analysis shows that the material structure of the windmill turbine won't fail due to repeated loading of magnitudes found from the CFD analysis. Thus it has a theoretical lifetime of infinity.

## 9. CONCLUSION

A doubly tapered and twisted beam was modeled and its natural frequencies were computationally derived. The values of natural frequencies were validated with a published research paper [7] and good agreement was found between the values suggesting that the modeling and analysis of the beam is correct. Then a real world model of a windmill blade was modeled and its natural frequencies were computed computationally using CAE software. These results were validated against the work of Perkins and Cromack [8] from which geometric parameters of the blade model had been referred. Again, the modal frequency values showed good agreement with the reference and the geometric model was validated along with the CAE analysis. A new attempt was made to calculate the lift and drag forces on the blade surface using CFD tools. These force values were used to carry out a stress analysis of the blade model and thus regions of high stress concentration were observed from the CAE results. The deformation along the blade span was also studied. Finally a fatigue life prediction analysis was implemented on the blade model by referring to S-N curves of used materials from various references. The fatigue result showed no failure for the model. Possible reasons could be the small size of the mini-model. Due to smaller span and shorter chord length, sufficient lift and drag forces could not have been generated that would cause failure of the material [18]. Usually windmill blades are 40 m in diameter [5] and over a few tons in mass, while this mini-model is just 24.49kgs in mass. Therefore even the effect of self-weight is almost negligible as far as deflection and induced stresses goes.

## 10. REFERENCES

- [1]. Yildirim, T. (2003) *Vibration analysis of pre-twisted rotating beams*. M.S. thesis, Izmir Institute of Technology, Turkey.
- [2]. Hassan, M. (2008) *Vibratory Analysis of Turbomachinery Blades*. M.S. thesis, Rensselaer Polytechnic Institute - Hartford, Connecticut.
- [3]. Hua, X.X., Sakiyamab, T., Matsudab, H, Morita, C. (2004). Fundamental vibration of rotating cantilever blades with pre-twist. *Journal of Sound and Vibration* 271 pp 47–66.
- [4]. Wikipedia [www.en.wikipedia.org](http://www.en.wikipedia.org)
- [5]. Johnson, S.J., Van Dam, C.P. and Berg, D.E. (2008). *Active Load Control Techniques for Wind Turbines*. Sandia National Laboratory, California, USA.
- [6]. Zehnder, A.T. and Warhaft, Z. (2011). University Collaboration on Wind Energy. Cornell University Atkinson Center for a Sustainable Future.
- [7]. Gupta, R.S. and Rao, S.S. (1978). Finite element eigenvalue analysis of tapered and twisted Timoshenko beams. *Journal of Sound and Vibration* 56(2), pp 187-200.
- [8]. Perkins, F.W. and Cromack, D.E. (1978). Wind Turbine Blade Stress Analysis and Natural Frequencies. Wind Energy Center Reports, Paper- 11.
- [9]. Changduk, K., Kim, T., Han, D., Sugiyama, Y. (2006). Investigation of fatigue life for a medium scale composite wind turbine blade. *International Journal of Fatigue* 28 pp 1382–1388
- [10]. Gunner, C.L., Morten, H.H, Baumgart, N.A., Carlén, I. (2002). *Modal Analysis of Wind Turbine Blades*. M.S. thesis, Riso National Laboratory, Roskilde, Denmark.
- [11]. Ganesan, R., Zabiollah, A. (2007) Vibration analysis of tapered composite beams using a higher-order finite element. Part I: Formulation. *Composite Structures* 77 pp 306–318.



- [12]. Ganesan, R., Zabihollah, A. (2007). Vibration analysis of tapered composite beams using a higher-order finite element. Part II: parametric study. *Composite Structures* 77 pp 319–330.
- [13]. He, K., Hoa, S.V. and Ganesan, R. (2000). The study of tapered laminated composite structures: a review. *Composites Science and Technology* 60 pp. 2643-2657.
- [14]. Gijs, A.M. and Van, K. (2007). The Lanchester–Betz–Joukowsky Limit. *Wind Energy*, Volume 10, pp. 289–291.
- [15]. Leishman, J. (2006). *Principles of Helicopter Aerodynamics*. 2nd edition Cambridge University Press, 2006. p. 751.
- [16]. Cottet, G.H. and Koumoutsakos, P. (2000). *Vortex Methods*. Cambridge University Press, p. 172.
- [17]. Leishman, J. (2006). *Principles of Helicopter Aerodynamics*. 2nd edition Cambridge University Press, p. 753.
- [18]. Holttinen, H. (2006). Design and Operation of Power Systems with Large Amounts of Wind Power. IEA Wind Summary Paper, Global Wind Power Conference, Adelaide, Australia.